Demand for Higher Education and Beliefs on Admission Chances

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Abstract

This paper studies the effect of changes to higher education supply on applicant demand in the Danish higher education market, which relies on a Deferred Acceptance type mechanism to match applicants and programs. To do this, I model the applicant's problem with a portfolio choice model and explicitly model applicants' subjective beliefs on admission chances. I estimate the model's parameters using detailed application data from 2014 combined with high-quality register data, allowing me to generate rich program characteristics. Having estimated the parameters of the model, I can then run counterfactual policy experiments where I reduce capacities for some programs to see how applicants respond. I find that, in general, applicant demand responds to changes in supply and that applicants to other programs than the directly affected also change their application behavior.

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1 Introduction

Worldwide, Centralized admission systems are widely used to assign students to schools and higher education programs[1]. Studies show that the allocation mechanism used by the Centralized Admission Systems (here on CAS) affects the welfare and other aspects of the allocations (Balinski and Sönmez, 1999; Abdulkadiroglu and Sönmez, 2003).

Much of this literature looks at how applicant demand responds to changes to the admission systems. In particular, many studies compare the applicant welfare under a manipulable allocation mechanism where applicants have a clear incentive to be strategic in their applications to a counterfactual strategy-proof mechanism, e.g., a Deferred Acceptance (here on DA) type algorithm similar to the one proposed by Gale and Shapley (1962). There are, however, few studies on how student demand responds to changes in supply. The answer is trivial when the mechanism is strategy-proof and applicants report their preferences truthfully. In that case, any changes in supply will only affect demand through changes in the characteristics of applicants and programs. Recent studies show that in many of the real world implementations of the deferred acceptance mechanism, with for example caps on the length of applications that can be submitted, the strategy-proofness result breaks down even though the mechanism itself is non-manipulative (Haeringer and Klijn, 2009; Hassidim et al., 2016; Fack et al., 2019; Artemov et al., 2020). It is not a priori clear how demand will respond to changes to supply when applicants do not report their preferences truthfully.

In this paper, I study how the demand for higher education responds to changes in supply, through changes in program capacities, in an admission system that uses a DA type algorithm where strategy-proofness fails. Further, I estimate applicant preferences for higher education programs. I base the estimation on Danish higher education application data combined with detailed administrative register data. The Danish CAS uses a DA type matching mechanism, similar to the one proposed by Gale and Shapley (1962) with some modifications. Most importantly, applicants cannot include more than eight programs in their applications and because of this constraint it is not necessarily optimal for applicants to report their preferences truthfully (Haeringer and Klijn, 2009; Hassidim et al., 2016; Fack et al., 2019; Artemov et al., 2020). In practice it means that applicants can choose to *skip the impossible* or *leave out not good enough* programs, such that the observed applications are a subset of the applicants true preferences. I report suggestive evidence showing that applicants to a large extent include programs which have GPA cutoffs close to their high school GPA and that they are reporting fewer programs than the limit of eight programs. If applicants reported their preferences truthfully I would expect them to include more programs and also to include programs with higher cutoffs as this should not affect their likelihood of receiving an offer from the other reported programs. The canonical model for applicant preferences assumes that applicants report their preferences truthfully, and estimates of

[1] I refer to major and university combinations as programs, e.g., economics at the University of Copenhagen.

preferences are therefore biased if this assumption fails. To avoid the mentioned bias and to be able to simulate counterfactual policy experiments where I change the supply through program capacities, I need to take the strategizing behavior of applicants into account.

To do this, I set up a portfolio choice model based on the framework in Chade and Smith (2006). The model allows me to relax the assumption of truth-telling by considering applicants' beliefs about assignment probabilities. This allows me to rationalize the observed application behaviour. I apply the bootstrap estimator suggested by Agarwal and Somaini (2018) combined with an assumption on rational beliefs on admission chances to estimate beliefs. I then combine these with detailed individual application data containing all applications in Denmark for 2014 and high-quality administrative register data to estimate applicant beliefs and preferences.

I further include a belief updating channel in my model. This in combination with assuming that preference parameters are policy invariant allows me to perform policy experiments to evaluate how applicant demand responds to changes in program capacities through applicants beliefs. In practice I use the portfolio choice model to solve for the optimal applications given a set of estimated parameters and re-estimating beliefs under a set of counterfactual capacities. I repeat these steps until beliefs have converged, and by comparing the simulated applications using the baseline beliefs with the simulated applications using the updated beliefs I can see how the model predicts demand will respond to a change in capacities. I compare the search patterns of students under the different policy experiments, and as I know the assignment mechanism, the final allocations of students for different capacities. In further also compare the final allocations in the policy experiments with a scenario where applicants do not have information on the changes to the capacities, and therefore are not able to update their beliefs. The changes in capacities that I consider are "neutral", as I redistribute any reductions in capacities for affected programs to unaffected programs according to the size of their prior capacities. The reason is that I only model the intensive margin of the application behavior (e.g., the decision for which program(s) to apply). As I do not model the extensive margin (e.g., whether to apply or not), the model is not well suited to measure the impact of an overall change in capacities.

I find that reducing capacities for programs in humanities while increasing capacities for other programs correspondingly causes applicants to change their application behavior, if they can update their beliefs. I see changes in application behavior even if I condition on whether applicants applied to programs before the change, who either have decreased or increased capacities now. However, applicants with an application to programs in humanities, before the change, shift away from humanities to a higher degree than other applicants shift away from their original field. Further, the characteristics of the programs that applicants with an application to programs in humanities, before the change, shift to are more similar to the programs of other applicants on average. Importantly I also find that revealing capacities and changes to capacities to applicants before they submit their applications might help applicants who otherwise end up being rejected.

This paper's contributions are multiple and relate to several different strands of the literature. Firstly, I contribute to the large and growing body of work on estimated preferences for higher education. Internationally many papers have estimated preferences for higher education, see for example Patnaik et al. (2021) for a recent survey of the literature. Particularly the contribution of this paper is estimating preferences based on rich measures of program characteristics generated from register data while relaxing the strong assumption of truthfulness by allowing applicants to be strategic in my model. Another way of relaxing the assumption of truthfulness is to use the estimator proposed by Fack et al. (2019). I do not use this approach as I need to fully model the application behaviour to be able perform the policy experiments. The body of work on estimating preferences for higher education is closely related to the literature studying school choice. The two problems are similar in many aspects although they differ in some important ways. Firstly schools mainly differ in the quality of the teaching and the distance to the students home, while university programs include a third dimension in the content which is taught (e.g. business or medicine). For university programs the third dimension is most likely the most important. Further, for university programs it is the student who makes the decision for which programs to apply for, while it is the parents of the student who make the decision in school choice. The seminal paper by Abdulkadiroglu and Sönmez (2003) framed the school choice problem as a mechanism design problem. They analyzed some of the existing allocation mechanisms and offer two alternative mechanisms which provide solutions to some existing problems. The paper inspired a large body of work to better understand the school choice problem and the effects of the allocation mechanisms. Agarwal and Somaini (2020) survey the recent methods to estimate preferences for schools and gives an overview of the empirical results.

Another branch of the literature on school choice is concerned with estimating preferences for schools or higher education programs when applicants do not report their preferences truthfully. Recently several papers have looked into this problem by looking at subjective beliefs on admission chances and information. Agarwal and Somaini (2020) study a model of school choice where applicants have beliefs of admission chances. In their main specification they assume that applicants have rational expectations when forming their beliefs. This approach assumes that applicants have full information when forming their beliefs. I use the same approach to estimate beliefs in my paper. Kapor et al. (2020) elicit beliefs through a survey and find that the beliefs are not in line with an assumption of rational expectations. They relax the assumption of rational expectations by allowing applicants to make mistakes when forming their beliefs. My current model does not allow me to consider mistakes in the formation of beliefs.[2]

[2]If applicants make mistakes when forming their beliefs it can bias my preference estimates, although it is difficult to say without having elicited beliefs.

Chen and He (2021, 2022) show that information costs can affect which preferences applicants report as they might not have enough information on all schools/programs and acquiring information can be costly. I do not properly model information costs in my model, but include a fixed information cost from including additional programs in the application. The reason is that it is difficult to separate the effect of beliefs and information costs on reported preferences.

Secondly, I contribute to the body of work which structurally models applicants' behavior in the school choice problem. Some other papers who model the applicants decision based on the portfolio choice model by Chade and Smith (2006) are Larroucau and Rios (2020), Larroucau and Rios (2022), Ekbatani (2022), and Ajayi and Sidibe (2015). Larroucau and Rios (2020), Larroucau and Rios (2022), and Ekbatani (2022) also model applications to higher education programs in Chile and Iran, while Ajayi and Sidibe (2015) set up at portfolio choice model to study school choice in Ghana. Calsamiglia et al. (2017) set up a structural model which they solve by backward induction for school choice in Barcelona.

Thirdly, I contribute to the small body of work on the effects of changes to the supply of higher education programs, through changes to capacities. My contribution is developing a model framework that allows the researcher to understand how the demand for higher education programs responds to intensive margin changes to the supply through changing capacities in a centralized application system using a deferred acceptance type or similar matching mechanisms. Another paper which studies this is Gandil (2022), who estimates the effect of supply changes on potential earnings, while holding preferences fixed. Gandil (2022) finds that changing supply with 10 slots leads to 15 applicants moving and that it explains 40 percent of the variation in earnings. This body of work is also related to the recent papers on field of study and earnings. Kirkeboen et al. (2016) exploit the implied randomness which arises close the the GPA cutoffs in university allocation mechanism which use a DA type mechanism, to identify the causal effect of crossing the threshold on earnings. Daly et al. (2022) build on this by looking at the effect of having first and second ranked programs in the same broad field versus having these in different broad fields. They find that only students with the first and second ranked programs in different broad fields, are negatively affected by not being accepted in their first ranked program.

The closest papers to this paper are Gandil (2022) and Larroucau and Rios (2020). While Gandil (2022) studies substitution effects that arise due to changes in program capacities for the Danish higher education market, as I do, my paper differs in some important ways. Firstly, he assumes that applicants report their preferences truthfully and that they are not affected by supply, such as e.g. Abdulkadiroglu and Sönmez (2003) and Azevedo and Leshno (2016). I relax the assumption on applicants reporting their preferences truthfully by explicitly modelling the applicants' behavior. His approach allows him to look at the substitution effects from changes to capacities for the whole application market, while I am currently limited to looking at a subsample of the pool of applicants as I do not estimate the preferences for the full population of applicants. Secondly, his focus is on showing the effect on foregone expected earnings caused by changes to supply, while my focus is on understanding how applicant demand responds to changes in supply. My scenario, where I do not allow applicants to update their beliefs, is equivalent to his approach. If I had estimated preferences for the full population, my approach could therefore be seen as containing his approach as a special case. Larroucau and Rios (2020) also develop a Portfolio Choice model to estimate applicant preferences for higher education programs. They do this for the Chilean higher education market and compare the estimates from their model to estimates obtained from a model which assumes strict truth-telling and find that these are biased. I use the same approach as they do to model the applicants' preferences. However, my approach differs as I adapt the model by adding a framework that allows applicants to update their beliefs in response to changes in supply. The mechanism I have implemented for updating applicant beliefs is similar to the mechanism in Larroucau and Rios (2022) but differs in its focus. They use it to update applicants' beliefs about their abilities in a dynamic setting, whereas I use it to update applicants' beliefs after changing capacities.

The remainder of the paper is organized as follows. Section 2 describes the Institutional settings and, in particular, the application system in Denmark. Section 3 describes the different data sources I use. Section 4 describes the application behavior and the characteristics of the programs in the observed applications. In section 5, I set up the Portfolio Choice model. Section 6 lays out the identification strategy. In section 7, I describe the approaches used to estimate beliefs and preference parameters. Section 8 presents and discusses the results and policy experiments. Finally, section 10 concludes with policy recommendations and suggestions for future research.

2 Institutional settings

In this section, I describe Denmark's higher education system, the centralized admission system, and the mechanism used to assign applicants to higher education programs.

2.1 Higher Education System

The higher education system in Denmark covers all educations post high-school degrees, e.g., education degrees offered by universities and business academies.

All higher education in Denmark is free, and students receive a stipend while enrolled.[3] Further, students in higher education in Denmark are also eligible for cheap student loans, where they are allowed to uptake a new loan corresponding to the yearly

[3]Students can receive it up to the normalized study time, e.g., five years for economics and six years for medicine, plus one additional year. The stipend comes with some conditions, e.g., how many ECTS points students have to be enrolled in yearly, although these conditions have changed over the years. In 2014 the stipend amount was 5, 839 DKK before taxes.

stipend each year with a very favorable repayment scheme. Students also receive a higher tax deduction.

Contrary to the higher education system in, e.g., the US, the Danish higher education system requires students to choose a major when applying, and most courses, except for some possible electives, will be within the subject of the chosen major. However, most programs also allow students to take courses offered by other programs, although there are often strict program-specific rules in place for the contents of these courses. Further, most students who graduate with a bachelor's degree also take a master's degree, according to a report by DST (2016) 83% of students who graduated with a bachelors degree in 2016 chose to enroll in a master's degree program in the same year.

2.2 Application system and mechanism

The CAS in Denmark handles virtually all applications for higher education. Applicants submit a Rank Ordered List (here on ROL) with their preferred programs to the CAS, which uses a student proposing DA type mechanism to match applicants and programs. By DA type mechanism, I mean that the mechanism is similar to the one proposed by Gale and Shapley (1962), with some important modifications. The first is an upper limit on how many programs each applicant can include in her application, and it is very common in real-world implementations of the DA mechanism. For example, in Denmark, the limit for the length of applications is set at 8 programs. In other countries, it typically does not vary much from this number with some outlier countries, e.g., it is 10 in Norway (Kirkebøen, 2012), 10 in Chile (Larroucau and Rios, 2020), while it is much higher in Iran, 100, (Ekbatani, 2022).

A second modification is splitting the program capacities into multiple quotas. In 2014, for example, the number of quotas was 70. The most used quotas are quota 1, quota 2, and quota 1 and 2 standby.[4] The applicants who apply for a given program through quota 1 are evaluated by their high school GPA and possibly some program-specific requirements, for example, a requirement to have passed a certain level of some set course in high school or a grade above some threshold in a certain course in high school or a program specific minimum GPA threshold. Quota 1 constitutes the majority of all applications as well as capacities (offers). Applicants who apply through quota 2 are evaluated on other measures than their high school GPA, e.g., grades or levels from specific courses in high school, motivational letters, relevant past work experience, and so forth, which differ across programs. It is, however, also important to note that all applicants who apply through quota 2 are first tested using the quota 1 criteria before being tested on the quota 2 criteria, if applicable (the student has a high school degree and the program's admission is not solely through quota 2). Lastly, applicants can also cross off the standby (either quota 1 or quota 2) option in combination with either

[4]The other quotas are, e.g., international students from non-Scandinavian countries and applicants from Greenland, among others. These make up an insignificant number of applicants and capacities, and I ignore them to simplify the analysis.

quota 1 or quota 2. Programs where the applicant has marked standby, are evaluated on the same requirements as pure quota 1 or 2. If an applicant is rejected on either of these "main" quotas for a given program the applicant is evaluated on the standby requirements for the standby capacity for the given program. If the applicant is offered a standby seat in a program, she is offered admission if enough applicants reject their offer. If not, she is guaranteed admission to this program next year. Being accepted in standby, however, also means that the applicant is not evaluated for any programs she listed as a lower priority in her application.

Most programs have quota 2 capacities ranging up to 10% of their total capacities. Further, the number of seats allocated to standby on quota 1 and 2 is also small compared to the total capacities.

Deadlines for applications differ by type of quota. For example, the deadline for submitting through quota 2 is on the 15th of March, and the deadline for submitting through quota 1 is on the 5th of July.

After all offers are given, applicants choose to accept or reject the offer. After that, the aftermarket begins. Applicants who were accepted through the standby quota are offered seats in the given program if rejected offers have freed up any. After that, applicants who were either not given an offer or rejected their offer and new applicants can apply for all programs with free capacities. The applications through the aftermarket go directly to the different programs. I ignore the aftermarket in this paper.

3 Data

In this section, I outline the different data sources used, detail how I generate the program-specific characteristics, and state the selection criteria used to define the samples used in the analysis.

3.1 Application data, capacities, and specific requirements

The primary data source is detailed information on submitted higher education applications in Denmark. I have access to all submitted applications to the Centralized Admissions System in Denmark for 1993-2015. This data contains demographic and educational background information on all applicants (GPA used for application, age, sex, citizenship, high school, type of high school, and other relevant information) along with information on which programs and, importantly, in what specific order each applicant has included these programs in her application and which program she received an offer from if any. The GPA measure in the application data includes GPA multipliers.[5] The application data also contains detailed information for each ranked program on

[5]Applicants in Denmark can multiply their GPA up before they apply by some multipliers based on, e.g., the number of A level courses in high school and if the applicant graduated from high school no more than two years prior to the application.

through which quota the application is. The raw dataset contains 3,385,555 applicants × program observations. I restrict the data to applications made in 2014, which leaves me with 244,198 applicants × program observations before I make any selection. There are three distinct reasons for only using applications in 2014. Firstly, I want data that is recent to be able to inform current policy. Secondly, I chose 2014 as I need a lot of data on graduated students to generate some program-specific characteristics, and it should ideally be data on students from earlier cohorts. Lastly, I chose 2014 as it is the last year before the Vacancy Based Dimensioning reform was implemented in 2015, and this allows me to relate the policy experiments to the reform.

The application data does not contain information on the capacities of programs in a given year. The information on program capacities is only publicly available for full programs. I was generously allowed access to this information from the Ministry of Higher Education and Science. The program capacities data contains information, for each program and year, on the capacities by to the different enrolment channels and the number of filled seats. Some programs have unrealistically high capacities, e.g., 999, and these should be interpreted as being higher than the number of filled seats, although the exact number is unknown. As I need these for my belief estimation and policy experiments, I need a more realistic measure of the capacity for these programs. My solution is to use the highest observed capacity for a given program across all available years. I will refine the solution to this problem in a future version of the paper.

Finally, I get information on program-specific requirements from the historic executive orders concerning admission requirements to the higher education programs in Denmark.[6] The program-specific requirements are admission requirements that are additional to the regular admission requirements and vary by program, and I have collected them specifically for this project. In particular, the program-specific requirements data contains requirements such as minimum GPA, minimum requirements for grades in certain high school courses, and minimum requirements for the level of certain high school courses.

Based on the application data, I create one of the program-specific characteristics I use to estimate applicants' preferences. The measure is the difference between an applicant's GPA in 2014 and the average GPA of accepted applicants in a given program in 2013, scaled by the standard deviation of the GPA of accepted applicants in the same program in 2013. I calculate the measure as

$$G_{ij} = \frac{GPA_{i,2014} - \overline{GPA}_{j,2013}}{\sigma_{GPA_{j2013}}}$$

where GPA_i is applicant GPA, $\overline{GPA}_{j,2013}$ is the average GPA of accepted applicants for program j in 2013, $\sigma_{GPA_{j2013}}$ is the standard deviation of the GPA of accepted applicants for program j in 2013, and G_{ij} is the standardized GPA of applicant i.

[6]The historic and current executive orders can be found on `www.retsinformation.dk`.

3.2 Administrative Register Data and distance measure

I mainly rely on administrative register data to generate the program-specific characteristics. However, to link the program characteristics, generated based on the registry data, with the application data, I first need to generate links between the program identifier in the application data and the program identifier in the education registry. To generate the links, I rely on the student registry (KOTRE), which contains all educational spells. I outline some issues related to forming the links and the approach I use to overcome these in appendix A.

After I have generated the links between the program identifiers in the application data to the program identifiers in the registries, I can generate three additional program characteristics for the expected labor market conditions after the applicants have graduated to help me estimate applicants' preferences.

To generate my three measures of labor market conditions, expected unemployment, expected earnings, and dispersion of expected earnings, I first combine the spell data from the student registry (KOTRE) with the link between program identifiers in the application data and the student registry, to identify all students who enrolled in a relevant bachelors degree in 2002 or later. I then condition on graduating with a bachelor's degree and enrolling and graduating with a subsequent master's degree. I keep the graduation date, month, and year, along with the program link and the personal identifier for students who graduated with their master's degree between 2008-2016. I then combine this data with the monthly employer-employee registry (BFL) for 2008-2016 to generate the measures for expected labor market conditions. To get a measure of expected unemployment, I first calculate the number of days between the date of graduation and the first day in the employer-employee registry, where I record a student as being employed in a position equivalent to 75% of full-time. I then estimate expected unemployment, U, as the average number of days between graduation and the first recorded day of employment divided by 30 to get months. I rely on a similar approach to get my measures of expected earnings and the dispersion of expected earnings. Instead of using the graduation date, I use the month. I then deflate the monthly earnings during the 12 months after graduation and divide by 10.000. Then I estimate the mean monthly earnings to use as my measure of expected earnings, $\overline{w}$. Finally, I estimate the dispersion of the expected earnings, σ_w as the standard deviation of the monthly earnings.

The distance measure, D, is created for this project by Denmark Statistics. It measures the road distance from the addresses of all high schools in Denmark to the main campus of all universities, such that the measure varies both by program and individual.

3.3 Sample selection

The full dataset contains information on 91, 276 applicants who apply to 897 different programs. After initial cleaning of the data, which mostly consists of removing canceled applications and applicants with multiple accepts, missing personal identifiers, and so on, results in a removal of 14, 741 applicants, leaving me with 80, 112 applicants, which I refer to as the *universe of applicants*. I use this sample to describe the overall characteristics of all applicants.

I select two samples from this *universe of applicants* to use in my estimation. I refer to them as the *full sample* and the *analysis sample*, which is a subset of the *full sample*. I use the *full sample* every time I run the allocation algorithm, which is a part of the estimation of subjective beliefs and the policy experiments. I use the *analysis sample* to estimate preferences and to summarize the results for the policy experiments.

To get the *full sample*, I remove all applicants with a missing high school GPA and applications to programs with zero capacities. This selection reduces the sample to 65, 214 applicants who apply to 693 different programs. As mentioned, I use this sample to estimate beliefs. The applicants with missing high school GPAs and applications to programs with zero capacities strictly apply through the quota 2 channel. As they are not tried on the quota 1 criteria, removing them does not affect the belief estimation.

To get the *analysis sample*, I select only applicants with applications strictly through the quota 1 channel, consisting only of university bachelor degrees. Further, I restrict the sample to applicants with an observable high school degree from a Danish high school. Lastly, I restrict the sample to applicants who only include programs with an observable value for all program and program/applicant characteristics used in the estimation (D, G, U, $\overline{w}$, and σ_w). This results in a sample of 12, 964 applicants with 24, 025 applications applying to 234 different programs. The two first selection requirements (quota 1 and bachelor) are highly correlated and the most important in terms of the reduction in sample size both by themselves and together.

4 Descriptives

In this section, I describe the overall characteristics of the applicants in the three different samples, the characteristics of the programs I observe in the applications, and the observed application behavior.

4.1 Applicant characteristics

Table 1 shows overall sample characteristics for three different samples. The table contains three panels, one for each sample. The first column shows mean characteristics, the second column shows standard deviations, and the third column shows the sample percentage with a non-missing value for the variable. By comparing panel C, the *analysis sample*, with the other two panels, we see that the applicants in the *analysis*

Table 1: Summary statistics by sample

Panel A: Universe of applicants	Mean	Std	Share in sample
GPA	7.21	2.53	85.10%
Female	0.53	0.50	100%
Age	22.54	5.62	91.58%
No. priorities	2.40	1.71	100%
Accepted	0.80	0.40	100%
Accepted priority	1.27	0.77	80.33%
Contains university program	0.52	0.50	100%
Only university programs	0.41	0.49	100%
Only through Quota 1	0.45	0.50	100%
Contains Quota 2	0.48	0.50	100%
Observations	80,112		
Panel B: Full sample	Mean	Std	Share in sample
GPA	7.29	2.52	100%
Female	0.55	0.50	100%
Age	21.84	4.85	93.57%
No. priorities	2.50	1.73	100%
Accepted	0.78	0.41	100%
Accepted priority	1.31	0.82	78.08%
Contains university program	0.63	0.48	100%
Only university programs	0.51	0.50	100%
Only through Quota 1	0.57	0.50	100%
Contains Quota 2	0.43	0.50	100%
Observations	65,214		
Panel C: Analysis sample	Mean	Std	Share in sample
GPA	9.10	2.26	100%
Female	0.51	0.50	100%
Age	20.21	2.48	98.43%
No. priorities	1.85	1.21	100%
Accepted	0.87	0.33	100%
Accepted priority	1.11	0.45	87.26%
Contains university program	1.00	0.00	100%
Only university programs	1.00	0.00	100%
Only through Quota 1	1.00	0.00	100%
Contains Quota 2	0.00	0.00	100%
Observations	12,964		

Note: The table contains three panels, panel A is for the *universe of applicants*, panel B is for the *full sample*, and panel C is for the *analysis sample*. The first column contains means for the selected variables, the second contains standard deviations, and the third contains the share of the given sample with non-missing values for the variable in percentages.

sample, on average, have a higher GPA, are younger, submit applications with fewer programs, are more likely to be accepted to any program, and are accepted on a higher ranked program in their application conditional on being accepted. Further, we also see that the analysis sample only contains applicants whose applications consist of university programs and only apply through the quota 1 channel, as I condition on these in the sample selection. Programs evaluate applicants through the Quota 2 channel based on measures other than high school GPA, e.g., grades or levels from specific courses in high school, motivational letters, and relevant past work experience, and I, therefore, expect them to have a lower GPA and be older on average.

4.2 Program characteristics

Table 2: Characteristics of programs in applications by rank

	Rank 1 Mean (Std)	Rank 2 Mean (Std)	Rank 3 Mean (Std)
Distance (10 Km)	7.15	8.45	8.98
	(7.80)	(8.16)	(8.55)
Standardized GPA	-0.19	-0.17	-0.19
	(1.68)	(1.72)	(1.71)
Unemployment (Months)	4.55	4.46	4.55
	(2.70)	(2.66)	(2.77)
Expected earnings (10,000 DKK)	2.50	2.52	2.52
	(0.58)	(0.59)	(0.59)
Dispersion of Expected earnings	1.14	1.18	1.16
	(0.44)	(0.57)	(0.52)
Observations	12,964	5,965	2,956

Note: The reported numbers are means (standard deviations in parentheses). The first column reports variable names and units in parenthesis. The columns indicate for which rank in the applications the measures are. The number of observations shows the number of applicants with at least the given number of ranks in their application.

Tables 2 and 3 show the characteristics of the programs in the observed applications for ranks one to three. Since the number of students with longer applications is low, I only rely on the three first ranks to generate moments for the estimation. Table 2 shows the mean and standard deviation by rank 1 to 3 of the observed applications for the different program characteristics. Higher ranked programs are, on average, a shorter distance to applicants' high school, while all the other characteristics are evenly distributed across the top three ranks on average. Appendix table 13 shows mean characteristics for ranks four to eight. The pattern in the distance continues to the lower ranked programs. At the same time, programs ranked lower in the application than rank 3 have higher expected unemployment, lead to lower expected earnings, a lower dispersion in expected earnings, and also have students in the prior application year with lower GPAs.

Table 3: The distribution of programs in different fields and universities by rank

Field	Rank 1	Rank 2	Rank 3
Social science	0.12	0.13	0.14
Humanities	0.20	0.21	0.21
Health	0.05	0.03	0.03
Natural science	0.21	0.22	0.22
Engineering	0.06	0.04	0.04
Other Business	0.08	0.10	0.09
Education	0.01	0.01	0.01
Economics	0.03	0.03	0.03
Medicine	0.08	0.08	0.09
Law	0.08	0.07	0.08
Political science	0.03	0.03	0.03
Business	0.06	0.05	0.05
University	Rank 1	Rank 2	Rank 3
University of Copenhagen (KU)	0.37	0.33	0.36
Aarhus University (AU)	0.20	0.21	0.21
Copenhagen Business School (CBS)	0.08	0.11	0.11
Aalborg University (AAU)	0.12	0.09	0.08
University of Southern Denmark (SDU)	0.13	0.15	0.13
Roskilde University (RUC)	0.05	0.06	0.05
Technical University of Denmark (DTU)	0.05	0.05	0.05
IT University of Copenhagen (ITU)	0.01	0.01	0.01
Observations	12,964	5,965	2,956

Note: The table shows the share of applicants who have ranked a program in a given field (top panel) or university (bottom panel) for the first three ranks in their applications. The number of observations shows the number of applicants with at least the given number of ranks in their application.

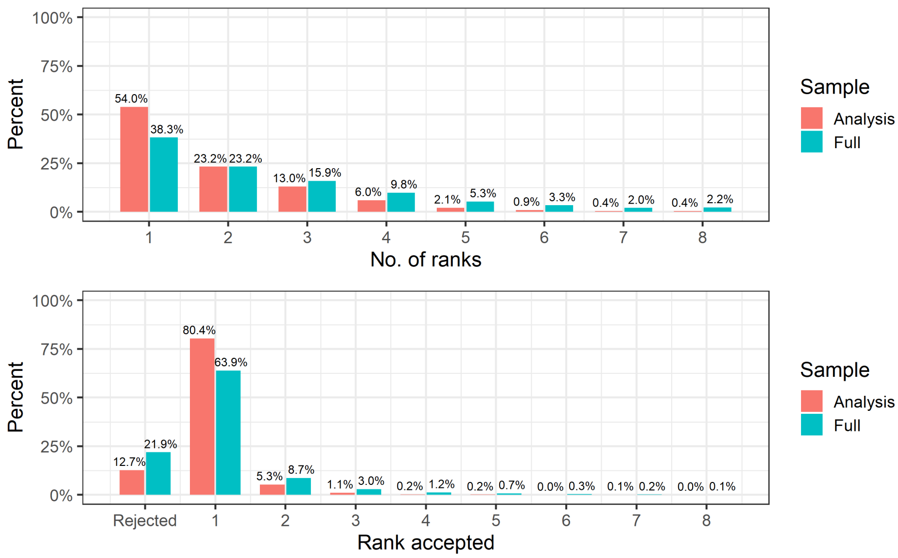
Table 3 shows the share of applicants who applied for a program in a given field[7] on the upper panel and a given university on the lower panel by ranks 1 to 3. Most applicants apply to a program in humanities as their top rank, with programs in social sciences (including economics and political science) in a close second place, followed

[7]Tabel 12 gives examples of which programs belong to the fields in table 3

by business (including other business and law). The patterns look similar for ranks 2 and 3 when conditioning on having at least two or three programs in the application. The pattern for universities is also quite similar across the top 3 ranks. Most applicants, 37%, have a program at the University of Copenhagen as their top rank, and the second most popular university is Aarhus University, with 20% of applicants choosing a program there as their top rank.

4.3 Application behaviour

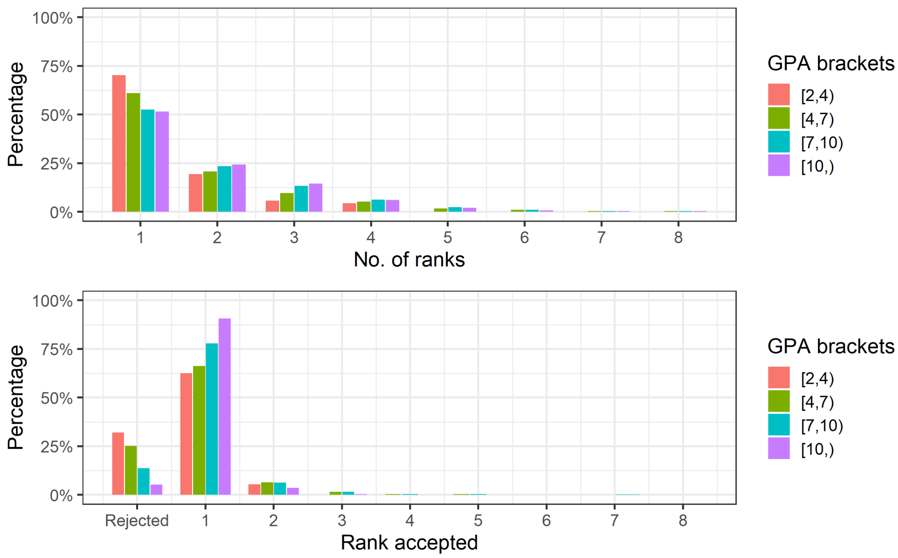
Figure 1: Application patterns for the full sample and the analysis sample



Note: The upper plot shows the share of applicants with a given ROL length, with percent on the second axis and ROL length on the first axis. The lower plot shows the distribution of accepted rank, if any.

Figure 1 shows applicants' overall application patterns in the *full sample* and the *analysis sample*. The upper panel shows the distribution of the length of the applications. Most applicants submit a short application with only one program, 54% of the *analysis sample*. The overall pattern is that as the length of applications increases, the share of applicants who have submitted an application of the given length falls. This pattern is clearer for the *analysis sample* compared to the *full sample*, with almost none of the applicants in the analysis sample submitting applications with five or more programs. The lower panel shows the unconditional distribution for on which rank applicants are accepted. We see that most applicants are accepted on their top ranked program and that this is more pronounced for the *analysis sample* (80.4%) compared to the *full sample* (63.9%). Further, we see that 12.7% of the *analysis sample* are rejected from all programs in their application, while this is 21.9% for the *full sample*. Further, we see that the share of applicants accepted on a given rank is falling with the number of ranks in the applications. The general pattern of short listing combined with the fact that low GPA applicants apply to programs with lower cutoffs the year before is in my interpretation a clear indication of strategic behaviour, where applicants with low GPAs top censor their applications, while applicants with high GPAs only include their most desired programs.

Figure 2: Application patterns for the full sample and the analysis sample by GPA

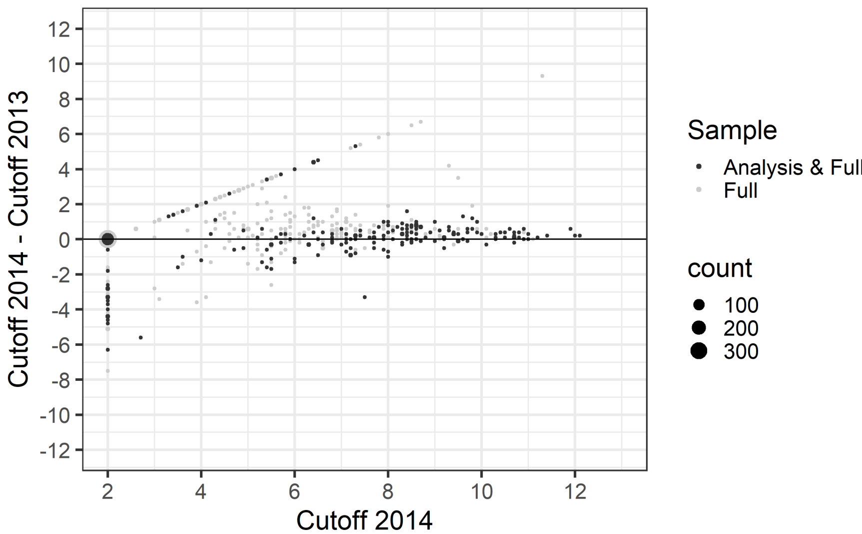


Note: The upper plot shows the distribution of the length of applications by GPA groups. The number of ranks in the applications is on the first axis, and shares in percentages are on the second axis. The lower plot shows the distribution for which rank an applicant is accepted on or if the applicant is rejected by GPA groups. For both plots, applicants are grouped by GPA accordingly 2 to 4, not including 4, 4 to 7, not including 7, 7 to 10, not including 10, and 10 or above.

Figure 2 shows the distribution of length of application in the top panel and which rank the applicant was accepted on, if any, on the bottom panel by GPA for the *analysis sample*. In the top panel, we see that applicants are almost equally likely to submit an application of a given length across the GPA distribution. If we turn to the bottom panel of figure 2 we see that applicants are also almost as likely to be accepted on a given priority across the GPA distribution. I interpret this finding as evidence of applicants not reporting their preferences truthfully. Almost no applicants reach the maximum length of their applications, so including one or more programs with historically higher GPA cutoffs on a higher rank would not affect their probability of being accepted in their current ranked programs.

Figure 3 shows the variation in cutoffs from 2013 to 2014. The first thing to notice is the two lines that the points sketch out. It is only feasible for points to be located

Figure 3: Change in cutoffs from 2013 to 2014



Note: The figure shows the change in cutoffs from 2013 to 2014. The first axis shows the cutoff in 2014, and the second axis shows the difference from 2013 to 2014. The number of programs scales the size of the points at a given point. The color of the points indicates whether a given program is only part of the *full sample* (grey) or part of the *analysis sample* as well (black).

on the lines or below the diagonal line and on the right of the vertical line. Programs on the bottom vertical line have the lowest possible cutoff in 2014, a GPA of 2, which means that the change from 2013 to 2014 can only be negative or zeros. Points on the top diagonal line had the lowest possible cutoff in 2013, a GPA of 2, so the resulting change will be equal to the cutoff in 2014 minus 2. The main takeaways from the figure are that there is much variation in yearly cutoffs and that many programs have non-binding cutoffs (although most are non-university programs). So applicants cannot predict the cutoffs perfectly. Remember, capacities and capacity changes are not public, and further students might not be able to predict how many other applicants there are or what their GPAs are.

5 Model

In this section, I formalize the applicants' education choice problem. In essence, the problem for the applicants is to optimally choose to submit a ROL that maximizes their expected utility. I base the framework on the optimal portfolio choice model in Chade and Smith (2006).

There are $i = 1, \ldots, N$ applicants choosing up to $\bar{R}$ out of J programs to include in their ROL, which they submit to the CAS. I denote the set holding all possible

ROLs, e.g., all combinations of programs for any given length of ROL up to $\bar{R}$, $\mathcal{R}$, and individual i's chosen ROL from all of the possible ROLs as R_i. The length of R_i, $|R_i|$, gives the number of ranked programs in R_i. All programs have positive capacities, $q_j > 0\,\forall j$. Further, all programs have identical and known preferences over applicants' scores.

All applicants have preferences over all j programs, sorted according to their indirect utilities u_{ij}, and all applicants have subjective beliefs about their admission chance to all j programs expressed by the vector given by $p_{ij} \in [0, 1]$. I further assume beliefs are independent across programs.

I can then formalize applicant i's expected utility from submitting a given ROL R_i as

$$\mathbb{E}U(R_i) = \sum_{k=1}^{|R_i|}\left(\prod_{r=1}^{r_k-1}(1-p_{ir})p_{ir_k}u_{ir_k}\right) + \prod_{k=1}^{|R_i|}(1-p_{ik})u_{i0}, \tag{1}$$

where u_{ij} is the indirect utility from being accepted to program j, p_{ir} is the individual and program-specific belief about the chance of being accepted to the program ranked as r. The k term in r_k describes the program's rank in the applicant's ROL. The last term is the value of the outside option. Applicants not accepted to any programs included in their ROL receive the value of the outside option, which I set to 0. Further, I require all applicants to list at least one program in their ROL, so my model does not capture the extensive margin fo the application behavior, e.g., whether to apply at all or not. The expected utility reflects the characteristics of the DA assignment mechanism in place, considering that an applicant will only be tried for admission at program j if she is rejected from all programs she ranked higher in her application.

The applicants' problem is then to maximize the following expression

$$\max_{R_i \in \mathcal{R}} \mathbb{E}U(R_i) - c(|R_i|), \tag{2}$$

where $\mathbb{E}U(R_i)$ is the expected utility from submitting ROL R_i and $c(|R_i|)$ is the application cost function, which depends on the length of the submitted ROL. As there are no pecuniary application costs, the cost function contains only non-pecuniary application costs, such as the information cost associated with finding programs to include in the application. I assume the cost function is linear $c(|R_i|) = |R_i| \cdot c$, where c is a small and fixed application cost. Further, I let the application cost function go to infinity for $|R_i| > 8$ as applicants cannot submit more than 8 programs in their applications. Finally, I calibrate the fixed cost c to $1e - 6$ to ensure that applicants only include programs in their application that strictly improve the expected utility. Otherwise, it is always optimal for applicants to include programs with a subjective probability of 0 in their applications as long as they have ranked fewer than eight programs.

Applicants then sequentially add programs, which increase the expected utility from submitting a given ROL, taking into account the cost of submitting the ROL. Applicants can either include programs in the top, the middle, or the bottom of their ROL, and Haeringer and Klijn (2009) show that it is optimal for applicants to sort the programs in their chosen ROL by expost utilities.

5.1 Indirect utility

The indirect utility applicant i receives from being accepted to program j is given by

$$u_{ij} = \alpha_j^F + \alpha_j^U + Z_j^P \alpha + Z_{ij}^S \beta + \varepsilon_{ij} \tag{3}$$

where α_j^F and α_j^U are field and university fixed effects, Z_j^P is a matrix containing program-specific characteristics given by

$$z_j^P = U_j \alpha_1 + \overline{w}_j \alpha_2 + \sigma_j^w \alpha_3, \tag{4}$$

where U_j is the expected unemployment, $\overline{w}_j$ is the expected earnings, and σ_j^w is the dispersion of expected earnings. Further, Z_{ij}^S is a matrix containing characteristics that vary across applicants and programs

$$z_{ij}^S = d_{ij} \beta_1 + G_{ij} \beta_2, \tag{5}$$

where d_{ij} is distance and G_{ij} is standardized GPA. Lastly, ε_{ij} is an additive idiosyncratic taste shock following a type I extreme value distribution.

5.2 Solving Portfolio Choice model

Solving the Portfolio Choice model requires finding the optimal portfolio from all possible portfolios for a given applicant. As the number of programs is $J = 234$ and applicants can rank up to $\bar{R} = 8$ programs in their application, this leads to $\binom{234}{8}$ possible combinations, which have to be evaluated for each applicant. Comparing all of them is infeasible. Instead, I use the Marginal Improvement Algorithm (MIA) proposed by Chade and Smith (2006) to find the optimal ROL for a given applicant. Chade and Smith (2006) show that if beliefs are independent and the cost function only depends on the length of the portfolio, it is only necessary to evaluate up to $\frac{234(234+1)}{2}$ portfolios and in most instances much fewer. The algorithm runs in the following steps

1 Start with $R_i^0 = \emptyset$

2 Choose $j = \max_{j \in J \setminus R_i^{n-1}} \mathbb{E}U(R_i^{n-1} \cup j)$

3 Set $R_i^n = R_i^{n-1} \cup j$ and order R_i by u_{ij}

4 Stop if $U(R_i^n) - U(R_i^{n-1}) < c(|R_i^n|) - c(|R_i^{n-1}|) = 0$ or all remaining $u_{ij} < u_{i0}$

In other words, the algorithm takes a given ROL, R_i, and calculates the expected value from including the remaining non-chosen alternatives in J one at a time. It thereafter checks which alternatives improve the expected value of submitting the ROL and chooses the one that gives the highest improvement in the expected value, if any. In other words, as the name suggests, the algorithm looks for the highest marginal improvement to the expected portfolio value, if any exists.

6 Identification

Identifying preferences for college programs is a challenging task. It is possible to rationalize every possible ordering by only looking at applicants' submitted ROLs, as it is impossible to separate strong preferences on unobservables from, e.g., beliefs. To identify applicants' preferences and beliefs separately, I rely on including what Agarwal and Somaini (2018) call a special regressor along with an assumption of rational expectations. The purpose of the special regressor is to include an exogenous variable, which only shifts preferences through indirect utility and not through beliefs. As Agarwal and Somaini (2018) suggest I include an individual-level measure of distance to education programs which I assume is exogenously determined. The specific distance measure I use is the road distance between applicant is high school and the location of program j, so my distance measure only varies for programs across universities and not within universities. This means that I also rely on the other measures in the indirect utility function, which vary across programs within universities as well, to separately identify preferences from beliefs.

Manski (2004) suggests that researchers circumvent the difficulties of separately identifying preferences and beliefs by eliciting subjective beliefs with, e.g., a survey. The approach suggested by Manski (2004) is not feasible in my case for two reasons. Firstly I rely on data on observed preferences, which was already collected, so I cannot go back and survey applicants. Secondly, my counterfactuals require me to be able to model applicants' belief formation. My approach is instead, to assume that applicants have rational expectations when they form their beliefs and that the beliefs can be estimated using the bootstrap estimator proposed in Agarwal and Somaini (2018).

To further help me identify the parameters of the indirect utility function, I use the variation in program characteristics and individual-program program characteristics in my data. I describe the different sample moments I am exploiting to identify the different parameters in section 7.

Changes in capacities are not announced prior to applications, and students can only infer changes to capacities for programs with a binding cutoff expost, conditional on the fact that the program had a binding cutoff in the prior year as well. I assume that changes to capacities happen exogenously from the student's perspective and that students do not base their application decisions on expectations for how program capacities might change from the previous year.

7 Estimation

In this section, I describe the estimation procedures used for the Portfolio Choice model and applicant beliefs.

7.1 Portfolio Choice model

The estimation of the portfolio choice model is not straightforward. As I mentioned in section 5, there are many possible portfolios and it is not feasible to solve for them all. Finding an expression for the choice probabilities would require me to find the expected value for all possible portfolios. As there is no convenient expression for the choice probabilities, I cannot use Maximum Likelihood estimation. What I do instead is to solve the model using MIA for some given applicant and program characteristics along with simulated taste shocks. I can then use the Simulated Method of Moments (SMM) for estimation. The SMM method implies simulating applications for a given guess on parameters and comparing the chosen moments from the simulated data with the same moments from the observed data and, in essence, minimizing the weighted distance between these. The SMM objective function is given by

$$Q(\theta) = (M_{simulated}(\theta) - M_{data})'\Omega^{-1}(M_{simulated}(\theta) - M_{data}), \qquad (6)$$

where M_{data} is a vector of moment conditions from the data and $M_{simulated}(\cdot)$ is the corresponding moments based on simulated data from the model for a given guess on the parameters θ. Ω is a weighting matrix. The initial estimation uses the identity matrix as the weighting matrix. Although the estimator is consistent with large N and a fixed number of simulations S, it will most likely be inefficient as it gives equal weight to all moment conditions, even though I do not expect that all moments are equally informative. The current choice of weighting matrix is a result of time constraints, as the model is computationally costly to solve and simulate. Later implementations will use a more efficient weighting matrix.

I select the moments to include based on the features I want the model to capture in the data. I first discretize all the continuous distributions for program characteristics (distance, standardized GPA, expected unemployment, expected earnings, and the dispersion of expected earnings) based on the application data. By using some discretization points I can calculate the empirical and simulated share of applicants within a given group for a given characteristic by, e.g., rank in their application, field, and university. I interpret the calculated shares as probabilities, and the estimation procedure then minimizes the distance between the empirical and simulated probabilities, where I first take the average over the simulations for the simulated moments.

I estimate moments in M_{data} which are based on the continuous measures with the following function

$$M_{kr} = \frac{\sum_i^{N_r} \mathbb{1}_{\{cut_{k-1} < variable_{ir} \geq cut_k\}}}{N_r}$$

where cut_k is the kth discretization point for a given variable, N_r is the number of applicants who have any program ranked as rank r, $\mathbb{1}$ is in indicator function, $variable_{ir}$ is the value of a given program characteristic for the program ranked on rank r by individual i. Hence, M_{kr} is the share of applicants out of the N_r, who have ranked programs on rank r with values of a given program characteristics within the interval between cut_{k-1} to cut_k. Table 4 displays my chosen discretization points.

Table 4: Discretization points for the moments

Measure	Cutpoint (cut_k)
D	$0 < 5 < 15 < \infty$
G	$-\infty < 0 < \infty$
U	$0 < 6 < 12 < \infty$
$\overline{w}$	$0 < 2.5 < 3.5 < \infty$
σ_w	$0 < 1 < 1.5 < \infty$

Note: The table reports the discretization points used to generate the moments for the simulated method of moments estimation.

In the estimation I use the discretized distributions of the measures directly and also include the share of applicants with any program on a given rank, as well as the share of applicants who rank a program in a given field or university and moments based on discretized measures of the continuous variables, which I have interacted with field and university dummies. I further only use the mentioned moments for ranks 1-3 and only include moments that are non zero for the observed data. This gives me a total of 781 moments.

In practice I find the reported parameter estimates in two steps. First I hand calibrate the parameters to values that seem close to a minimum for the criterion function, thereafter I feed these calibrated parameters as initial values to the 'fminsearch' minimizer in Matlab which uses the Nelder-Mead simplex algorithm to estimate the parameters.

I have not estimated standard errors for the estimated parameters in the current estimation framework. Estimating standard errors requires me to estimate the variance-covariance matrix for the moments, which is currently too computationally costly to implement for this paper version. I will estimate standard errors in a future version of the paper.

7.2 Beliefs

I do not have information on subjective beliefs, but I can estimate them. Agarwal and Somaini (2018) show that under an assumption of rational expectations, it is possible to estimate subjective beliefs consistently using their bootstrap estimator. Their proposed method samples applicants and their applications with replacement from the population of applicants and runs the assignment algorithm to get the cutoffs for each sample. Repeating this many times in a bootstrap routine makes it possible to characterize the distribution of cutoffs. To estimate the individual subjective belief for a program, I calculate the fraction of times individual i's GPA is equal to or above the simulated cutoff for program j. The method requires information on all applicants and their applications and admission scores, as well as information on the mechanism used to match applicants with programs and the capacities of the programs. The bootstrap estimator is given by the following expression

$$\hat{p}_{ij} = \frac{1}{B}\sum_{b=1}^{B} \mathbb{1}_{\{s_i \geq P_{jb}\}} \tag{7}$$

where $\hat{p}_{ij}$ is the estimated beliefs, $\mathbb{1}$ is an indicator function that is equal to one when the students score, s_i, is greater than or equal to the simulated cutoff. P_{jb} is the simulated cutoff for program j in bootstrap simulation b. I only estimate beliefs for the *analysis sample*, although I need the *full sample* to simulate the cutoff distributions for the programs. The estimation results in a N-by-J matrix which contains the estimated beliefs, $\hat{p}_{ij}$, in each cell.

As mentioned in section 4 I draw from the pool of all applications to simulate the different cutoffs, while I only use the *analysis sample* to estimate the beliefs. The reason for doing this is that it is impossible to subset the capacities in a meaningful way, and simulating the cutoffs on a sub-sample of the pool of applicants would result in many simulations where many programs never reach their capacities.

As mentioned in section 2, the Danish assignment procedure differs slightly from the standard DA mechanism (mainly as it includes different quotas, standby applications, and applicants cannot submit more than 8 programs). However, knowledge about the allocation mechanism allows me to consider these to get consistent estimates of the beliefs.

Firstly the option to apply for quota 1 standby is trivial to incorporate as it follows the same GPA ranking as quota 1. It is thereby possible to allow for additional standby seats, which only students applying on standby can fill, resulting in additional standby cutoffs for each program with this application channel. It is more difficult to incorporate the quota 2 and quota 2 standby channels. This is because the assignment criteria are unknown/nontransparent and program-specific. The different programs rank applicants based on some point system, which partly relies on subjective evaluations and changes from program to program. This is because the CAS needs a ranking of the applicants to allocate them. Unfortunately, I do not have access to data on these rankings.[8] If I choose not to include quota 2 applicants in my analysis, I can take advantage of the fact that all quota 2 applications are first tested using quota 1 criteria. This implies that $p_{q1} \leq p_{q2}$ or the individual probability of acceptance for quota 2 applicants has a lower bound, which is the individual probability of acceptance through the quota 1 channel. Further, as I am only interested in estimating preferences for quota 1 applicants, I can use the knowledge of whether quota 2 applicants were accepted for a given program through the quota 2 channel as a measure of the quality of their quota 2 application. What I do in practice is that I observe for which programs applicants are accepted on quota 2. If an applicant reaches such a program in the mechanism and is rejected on the quota 1 criteria, I accept the applicant in quota 2.

This approach to estimating beliefs is crucial for conducting my policy experiment, as it requires explicit modeling of the belief formation of applicants, as this is the only channel through which capacities can affect application behavior.

8 Results

In this section, I describe the results. I first describe the estimated utility parameters and university and field fixed effects before I validate how well the model fits the observed data.

8.1 Estimated preference parameters

Table 5 shows the estimated utility parameters for the Portfolio Choice model. Applicants prefer programs close to the high school they attended, where the accepted applicants in the prior year had higher GPAs on average, with lower expected unemployment, higher expected earnings, and a higher dispersion in expected earnings. The estimated coefficient on distance is as expected negative, such that applicants prefer closer programs, proxied by the location of their high school, to programs further away. The negative sign on the coefficient on G means that applicants prefer programs where they have a lower GPA than the average accepted applicant in the previous year. This is in line with a story of applicants not wanting to waste their GPAs and therefore apply to programs with higher GPA requirements. However, the interpretation comes with a limit as applicants are also more likely to rank programs where they have a higher belief about being accepted, which requires a relatively high grade compared to the other applicants in the current application year. As the parameter on U is negative, applicants prefer programs with lower expected unemployment. Further, they also prefer programs with higher expected earnings and programs with a higher dispersion of expected earnings. Hence applicants prefer programs with better labor market prospects, e.g., low

[8] It would likely not help me much, even if I had access to the rankings, as my policy experiments would require me to model the quota 2 evaluation process.

Table 5: Estimated parameters

Parameter	Value
D	-0.104
	(.)
G	-0.200
	(.)
U	-0.009
	(.)
$\overline{w}$	0.127
	(.)
σ_w	0.035
	(.)
Q	0.634
Simulations	2
Applicants	12,964
Observations	24,025

Note: The table reports parameter estimates and standard errors in parentheses. The current solution and estimation procedure make the estimation of standard errors too time consuming, so I have not estimated them. The table also contains the value of the minimized criterion function, Q, and the number of applicants and observations used in the estimation.

unemployment and high wages, with a higher chance of high wages.

Tables 6 and 7 show the estimated fixed effect parameters on field and university fixed effects. The field fixed effects are all in relation to the field *Social sciences, excl. Economics and Political Science* and the university fixed effects are in relation to the *University of Copenhagen*. We see that conditional on the other variables of the utility function, law, business, and medicine are the most preferred fields, followed by other business and health. Looking at the university fixed effects, we see that conditional on the other variables of the utility function, Roskilde university and Aalborg University are slightly more preferred than the University of Copenhagen, while Copenhagen Business School, the Technical University of Denmark, and the IT University of Copenhagen are the least preferred.

Table 6: Estimated field fixed effect parameters

Parameter	Value
Social science	ref.
	(.)
Humanities	-0.132
	(.)
Health	0.652
	(.)
Natural science	-0.126
	(.)
Engineering	-0.211
	(.)
Other Business	0.618
	(.)
Education	-0.051
	(.)
Economics	0.076
	(.)
Medicine	3.539
	(.)
Law	2.306
	(.)
Political science	-0.894
	(.)
Business	0.955
	(.)

Note: The table reports the estimated parameters for the field dummies. The reference category is Social science.

8.2 Validation of Portfolio Choice model

To validate the Portfolio Choice model, I make two comparisons. Firstly, I check how some of the moments used in the estimation compare with the simulated, and secondly, I check if the simulated data moments are close to hold-out moments. The first check is less demanding than the second check as the estimation procedure explicitly minimizes this difference. I have used the first three ranks of applications to fit the model. While applicants can rank up to eight different programs in their applications and I therefore

Table 7: Estimated university fixed effect parameters

Parameter	Value
University of Copenhagen	ref.
	(.)
Aarhus University	-0.131
	(.)
Copenhagen Business School	-0.351
	(.)
Aalborg University	0.203
	(.)
University of Southern Denmark	-0.060
	(.)
Roskilde University	0.115
	(.)
Technical University of Denmark	-0.438
	(.)
IT University of Copenhagen	-0.964
	(.)

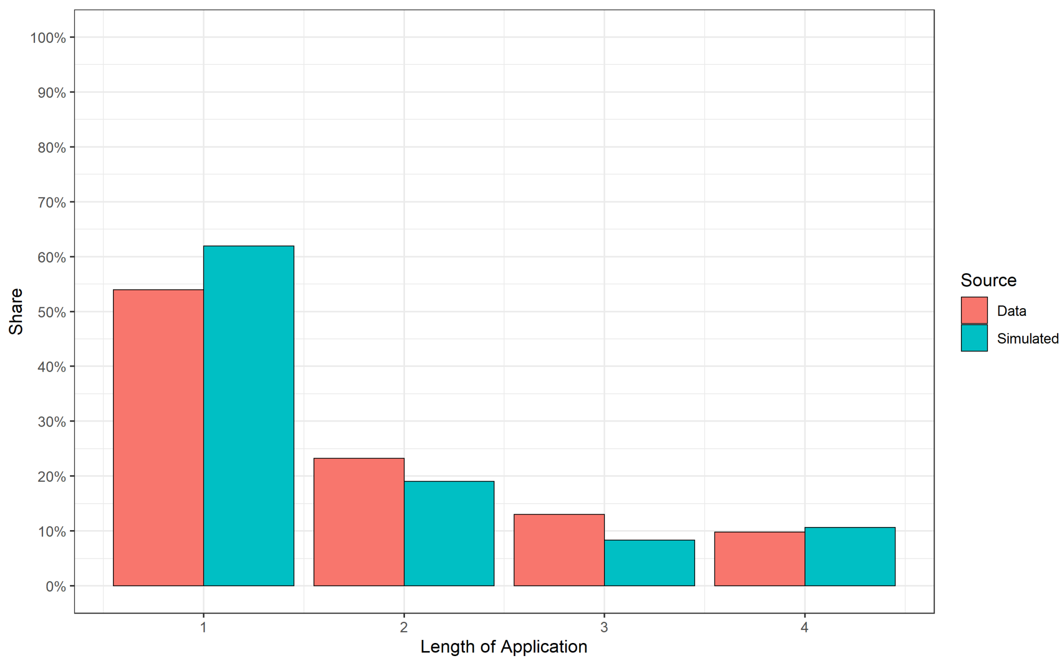
Note: The table reports the estimated parameters for the university dummies. The reference category is University of Copenhagen.

have potential hold out moments for five ranks, the share of applicants who submit an application of a given length is falling by the length, so I only use the moments for the fourth rank as hold out moments.

Figure 4 shows the distribution of the length of application for the observed applications in the *analysis sample* and the simulated applications from the model. We see that the model captures the overall characteristics of the distribution for length of application quite well, although it over-predicts the share of applicants with short applications (only one program) and under-predicts applicants with longer applications (more than one program). The model seems to capture the share of applicants with four programs in their application quite nicely as well.

Table 8 shows how well the model does in terms of capturing the shares of applicants who rank a program in a given field (Panel A), shares of applicants who rank a program in a given university (panel B), and the means of program characteristics over the four top ranks. The moments based on the first three ranks are used to fit the model. If we first focus on ranks one to three we see that the model overall captures the moments used in the estimation fairly well. Looking at panel A we see that the difference between the empirical and simulated shares for the different fields is in general below four percentage

Figure 4: Empirical and simulated share of applicants by the number of programs in their application



Note: The first axis displays the number of programs in an application (length of the application) and the second axis displays the share of applicants in percentages. The color of the bars indicates the source, red represents the empirical distribution and green indicates the simulated distribution.

Table 8: Comparison of data and simulated moments

Panel A: Field	Rank 1		Rank 2		Rank 3		Rank 4	
	Δ	%	Δ	%	Δ	%	Δ	%
Social science	0.039	32.99%	0.024	39.67%	0.011	33.96%	0.000	-3.50%
Humanities	0.005	2.26%	0.005	4.76%	-0.001	-3.03%	-0.007	-31.64%
Health	-0.012	-23.97%	-0.008	-53.33%	-0.003	-48.33%	-0.001	-27.00%
Natural science	0.032	15.29%	0.022	22.17%	0.005	9.49%	-0.008	-37.88%
Engineering	-0.008	-13.61%	-0.011	-62.67%	-0.008	-89.45%	-0.006	-215.79%
Other Business	-0.023	-30.11%	0.006	13.19%	0.000	1.82%	-0.004	-50.00%
Education	-0.008	-100.00%	-0.004	-138.57%	-0.001	-81.82%	-0.001	-90.91%
Economics	0.014	45.10%	0.003	29.00%	0.001	20.51%	0.000	6.41%
Medicine	-0.072	-85.34%	0.017	44.54%	0.018	82.43%	0.010	96.38%
Law	-0.010	-12.60%	0.008	21.86%	0.010	56.09%	0.007	76.23%
Political science	0.021	82.14%	0.012	86.22%	0.006	90.96%	0.002	79.03%
Business	0.021	34.36%	0.004	20.65%	0.001	11.23%	-0.002	-52.50%
Panel B: University	Rank 1		Rank 2		Rank 3		Rank 4	
	Δ	%	Δ	%	Δ	%	Δ	%
KU	0.080	21.8%	0.039	25.7%	0.023	27.8%	0.000	-0.22%
AU	-0.024	-12.3%	0.011	11.8%	0.010	20.4%	-0.003	-16.89%
CBS	0.005	6.7%	0.019	39.0%	0.011	43.1%	0.001	9.50%
AAU	-0.041	-33.2%	-0.015	-33.5%	-0.007	-38.5%	0.002	11.20%
SDU	-0.053	-42.1%	0.003	5.1%	-0.004	-12.7%	-0.007	-52.73%
RUC	0.017	36.3%	0.011	44.0%	0.002	19.4%	-0.001	-29.69%
DTU	0.010	20.5%	0.008	34.5%	0.003	24.3%	0.000	-8.77%
ITU	0.007	62.2%	0.002	47.2%	0.000	23.5%	0.000	27.27%
Panel C: Program char.	Rank 1		Rank 2		Rank 3		Rank 4	
	Δ	%	Δ	%	Δ	%	Δ	%
Distance	-1.054	-14.7%	0.261	3.1%	0.658	7.3%	2.619	25.29%
G	0.003	-1.7%	0.702	-417.5%	1.164	-615.8%	1.265	-350.23%
U	0.332	7.3%	-0.109	-2.4%	-0.118	-2.6%	-0.204	-4.38%
Mearn	-0.070	-2.8%	0.041	1.6%	0.070	2.8%	0.080	3.24%
Stdearn	0.077	6.8%	0.094	8.0%	0.064	5.5%	0.015	1.36%

Note: The table reports differences between empirical and simulated moments (Δ) and the difference in percent of the empirical moment (%) for ranks one to four in the applications. The moments based on the first three ranks are used to fit the model, while the moments based on the fourth rank are hold out moments. Panel A shows how well the model captures the share of applicants who have a program in a given field on ranks one to four. Panel B shows how well the model captures the share of applicants with a program in a given university for ransk one to four. Panel C shows how well the model captures the mean program characteristics for rank one to four.

The field Social science is excluding Economics and Political science and the field Health is excluding Medicine.

points ($100 \cdot \Delta$) except for medicine on the top rank, where the model over predicts the share of applicants with a program in Medicine as their top priority by around seven percentage points. The get an understanding of the magnitude of the differences we can look at the columns with the differences in terms of percentages of the empirical share (%). We see that a Δ of 0.072 for Medicine on the top rank corresponds to 85.34% of the empirical share, meaning that the difference is almost as large as the empirical share. If we next move to panel B, we see that the model also does a fairly good job of capturing the shares of applicants with a program in a given university for the first three ranks. The largest difference is that the model under predicts the share of applicants with a program at University of Copenhagen (KU) as their top rank by eight percentage points, although as the share of applicants who rank programs at the University of Copenhagen in general is large, the difference only corresponds to 21.8% of the empirical share. Lastly, if we look at panel C we see that the model seems to capture all the program characteristics except G quite well.

To conclude the model fits estimation moments quite well. The more demanding test is however how the model fits the hold out moments (rank four). For panel A and B the model captures the shares for fields and universities as well and in some instances better than the estimation moments. In panel C the model does almost as good for the hold out moments as the moments used in the estimation, except for distance, where the model under predicts the mean distance to the programs included on the fourth rank by around 26 km ($\Delta \cdot 10$km), which corresponds to 25.29% of the empirical mean distance on the fourth rank.

9 Policy experiments

Having described applicant preferences with my model, I now turn to the next point of interest, simulating the effects of supply changes, through changes in capacities, under two different scenarios. Under the first scenario, applicants cannot update their beliefs, and under the second, they can update their beliefs. As the model only captures the demand side of the education market, I can only look at the effects in a partial equilibrium. Therefore, I will not be able to perform a full welfare analysis. I can, however, look at how the different policies affect the distributions of applications, outcomes, and the characteristics of offered programs.

The Portfolio Choice model allows me to perform policy experiments where I change the supply for programs, as the supply only affects the subjective applicant beliefs through the available capacities in the model. Program capacities are only public after all offers are given and only for programs with exhausted capacities. This means that applicants can only see a program's capacity in the previous years (if the capacity was exhausted) and cannot see the change in capacity from the previous year to the current year. While this might give applicants some information on the available capacities, at least for programs with binding cutoffs, it is limited how they can use it when forming their subjective beliefs, as they cannot take possible changes into account.

The section proceeds as follows. I first describe how the policy changes affect applicants in my model and the channel through which applicants can update their beliefs according to the policy changes. After that, I present and discuss the effects of the proposed reduction in capacities for programs within the field of humanities.

9.1 Changes to capacities and belief updating

The proposed policy experiments aim to understand how changes to capacities affect applicants' demand for programs. Capacities enter my model through the CAS matching mechanism. The capacities enter the applicants' expected utility from submitting a given portfolio in the subjective program-specific beliefs and further, also affect which program applicants are offered, if any. To evaluate the effect of the proposed policies, I need to know and be able to implement the allocation mechanism. Further, I need to include a channel through which applicants can update their beliefs according to changes in capacities.

I run the policy experiments in the following procedure. I first simulate the model for different capacities under the current framework, where the changes are unanticipated shocks to the capacities, and applicants cannot consider them when forming their beliefs. Second, I look at the same changes to capacities where I instead reveal them to applicants before they form their applications. This allows applicants to update their subjective beliefs according to the new capacities. In my model, program capacities only affect applicants' expected utility from submitting a given application through their subjective beliefs. The indirect utility derived from being admitted to a given program is unaffected. I can therefore find the effect of the proposed policy experiment by only varying capacities and whether I allow applicants to update their subjective beliefs or not while holding the policy invariant utility parameters fixed.

Applicants update their beliefs according to the algorithm in appendix C. The algorithm runs until the cutoff distributions for all programs converge, where each point in the distributions is the equilibrium cutoff from taking a sample with replacements from the *full sample* of applicants and running the allocation mechanism. In each iteration, I reestimate beliefs and solve the portfolio choice model for the given indirect utility parameters and the new estimated beliefs. When the euclidean norm over the vectors of means and standard deviations of the cutoff distributions are below the tolerance parameter ($\epsilon = 1e - 6$) I say the algorithm has converged.

Before I run the policy experiments, I need an additional step. The beliefs estimated from the data are based on the observed applications. So to get a baseline for the updating algorithm, which is aligned with the simulations from the model, I reestimate beliefs using the simulated applications from the model. I then use the new beliefs along with the preference parameters in the policy experiments. Figure 8 illustrates the new predicted shares of applicants with an application containing one to four programs. We see that it has changed very little compared to figure 4.

The specific changes I make to capacities are reductions to the capacities in programs within the field of humanities. This is interesting as the vacancy-based dimensioning reform mainly affects programs in the field of humanities.

As the current model does not include an outside option as a choice (I have constrained all applicants to submit at least one program in their applications), a reduction in overall capacities leads to the easily predictive result where fewer applicants are accepted. To avoid this scenario, I instead look at capacity-neutral changes. In other words, when I reduce capacities for some programs, e.g., within humanities, I redistribute the removed capacities among all other programs to keep the same overall capacities. There are many different ways to redistribute capacities, I choose to redistribute capacities according to the distribution of capacities on the program level for unaffected programs, using the baseline capacities. This means that if a program for example had 5% of total capacities excluding programs within humanities before I change capacities, this program will receive 5% of the total capacities that I remove from the programs within humanities.

Lastly, it is important to note, that as I rely on the *full sample* when I run the allocation mechanism, and I only estimate preferences for applicants in the *analysis sample*, the preferences for all other applicants in the *full sample* are fixed when I run the allocation mechanism to look at where applicants are accepted in the policy experiments. All the reported results for the policy experiments only contain the applicants in the *analysis sample*, but their application behavior is also affected by the applications of the other applicants, as the result from the allocation mechanism is a general equilibrium outcome based on the preferences of applicants and programs within the constraint of the capacities. In a future version of the paper I will look more closely at how this affects my results.

9.2 Reducing capacities for humanities

Table 9 shows the field level capacities for different values of the policy parameter γ. The capacities are for all programs included in at least one application for the *analysis sample*. The first column shows the baseline total program capacities by field, and the subsequent columns show how the capacities change by fields for different policy parameter values (γ). We see that the humanities programs' capacity drops as the value of γ falls while the program capacities in the other fields increase. This is a feature of the neutral policy design, where reductions in the capacities of programs within one field are redistributed across the programs in the other fields by the baseline program capacities of unaffected programs. We see that the field with the highest number of capacities in the baseline setting is humanities, with the field Natural science and the fields within business and law (Business, Other Business, and Law) in a close second and third place. The chosen redistribution rule means that I redistribute most of the cut capacities within humanities to programs within the mentioned broad fields.

Table 9: Capacities for the different policies

	$\gamma = 1$	$\gamma = 0.9$	$\gamma = 0.5$	$\gamma = 0.1$
Capacities				
Social science	2,694	2,776	3,099	3,431
Humanities	4,760	4,289	2,401	483
Health	962	992	1,106	1,223
Natural science	4,428	4,562	5,099	5,635
Engineering	1,014	1,047	1,167	1,290
Other Business	2,180	2,244	2,508	2,775
Education	241	249	276	307
Economics	563	579	647	716
Medicine	1,094	1,126	1,259	1,393
Law	1,292	1,330	1,487	1,644
Political science	296	305	340	376
Business	920	948	1,058	1,171
Total excl. Humanities	15,684	16,158	18,041	19,961
Total	20,444	20,447	20,442	20,444

Note: The policy variable γ, which indicates what share of the capacities in the programs within humanities are left, while I redistribute $1-\gamma$ capacities from the programs within humanities across the programs within the other fields. The capacities in the first column, where $\gamma = 1$ are the baseline observed capacities. Due to the rounding of changed capacities to integers on the program level, the sum of capacities for all columns is not the same.

9.3 Applications and program characteristics

Table 10: Policy changes to capacities: Applications

	Baseline	No updating			Updating		
γ (Fraction of original capacity)	1	0.9	0.5	0.1	0.9	0.5	0.1
Panel A: Humanities							
Length of application	1.98	0.00	0.00	0.00	0.08	0.23	0.07
Accepted	0.84	-0.01	-0.13	-0.49	0.01	-0.04	-0.08
Rank accepted on	1.11	0.03	0.20	0.52	0.01	0.09	0.15
Number of fields	1.54	0.00	0.00	0.00	0.07	0.27	0.23
Same top-ranked program	1.00	0.00	0.00	0.00	-0.08	-0.29	-0.67
Same top-ranked field	1.00	0.00	0.00	0.00	-0.06	-0.25	-0.65
Panel B: All other fields							
Length of application	1.78	0.00	0.00	0.00	-0.02	-0.11	-0.20
Accepted	0.83	0.01	0.03	0.05	0.02	0.03	0.03
Rank accepted on	1.13	-0.01	-0.06	-0.09	-0.03	-0.05	-0.06
Number of fields	1.52	0.00	0.00	0.00	-0.02	-0.06	-0.11
Same top-ranked program	1.00	0.00	0.00	0.00	-0.06	-0.04	-0.06
Same top-ranked field	1.00	0.00	0.00	0.00	-0.05	-0.03	-0.05
Observations	12,964						
Simulations	2						

Note: Column 1 (γ = 1) contains the baseline application characteristics and outcomes of the matching mechanism. Panel A is for applicants with a program in Humanities as their top rank in the baseline setting, and panel B is for applicants with other programs as their top rank in the baseline setting. The numbers of columns 2 to 7 are all expressed as row-wise deviations from column 1.

Table 10 shows the overall counterfactual characteristics of applications split up into two panels, panel A for applicants with a program within humanities as the top rank in their application with the original capacities and panel B for all other applicants for the different values of the policy variable γ and whether applicants can update their beliefs according to a change in program capacities. If applicants cannot update their beliefs, the submitted applications will stay the same, although the realized outcomes from the matching mechanism can change as the capacities change. The first column with $gamma = 1$ is the baseline setting, where the capacities equal the observed capacities. I express all counterfactual results, columns 2-7, as deviations from the results in the baseline setting. I have fixed the applicants for each panel in the table, so even though some applicants, e.g., change their top-ranked program from a program in humanities (with the original capacities) to a program in another field (with changed capacities), they are part of the results in panel A, and likewise for panel B.

I first compare the applications and the outcome of submitted applications for applicants with a program in humanities as their top priority (panel A) to other applicants (panel B) under the baseline setting ($\gamma = 1$). Overall, applicants with humanities programs as their top priority submit longer applications. At the same time, they are as likely to be accepted to a program and, conditional on being accepted, they are accepted on the same rank in their application as the other applicants. The last two categories are not interesting in the baseline setting, as they measure if the applicant has the same top-ranked program or field as in the baseline setting.

Next, I look at columns two to four, which show the deviation from the baseline setting for different policy parameter values under the no updating setting. We see that the length of applications and the shares of applicants with the same top-ranked program or field are the same as in column one. This is because applicants can only change their applications through changes in beliefs, and in the no updating setting, I do not allow applicants to update their beliefs. As expected, for columns 2-4 in panel A, we see that the share of applicants, who are accepted to any program, falls as I reduce the capacities for the programs in humanities, although the share of accepted applicants does not fall one to one with the reduction in capacities. We also see that the applicants are on average accepted at a lower ranked program in their applications. In columns 2-4 in panel B of table 10, we see the opposite pattern, although to a much lesser extent. This stems from the fact that the change in capacities is neutral in total capacities. Hence, programs in fields other than humanities get increasingly higher capacities as capacities within humanities are reduced, as we see in table 9.

Next, I turn to the setting where applicants can update their beliefs to take the changes to capacities into account, columns five to eight in table 10. By comparing panels A and B, we see that applicants with humanities as their top rank in the baseline setting increase the length of their applications, while the applicants with other fields as their top rank in the baseline setting decrease the length. Further, we see that applicants in panel A have a larger probability of being accepted for a small change in capacities ($\gamma = 0.9$), while they have a slightly lower probability of being accepted for larger changes to capacities ($\gamma < 0.9$). On the other hand, applicants in panel B have a slightly higher chance of being accepted, which increases with the change in capacities. For the measure of which rank the applicants are accepted on, we see that applicants in panel A are accepted to a lower ranked program in their applications, while applicants in panel B are accepted to a higher rank in their applications. Last, we see that applicants in panel A increase the number of fields in their applications and are less likely to rank the same program or field as their top rank when capacities in humanities decrease. For applicants in panel B we see that they include fewer fields in their applications as we change the capacities, while they are slightly less likely to report the same program or field as their top rank.

To sum up, the results from table 10 show that when I allow applicants to update their beliefs, applicants in panel A can change their applications such that their chance of being accepted is not affected nearly as much, e.g., 76% compared to 35% in the most extreme case with $\gamma = 0.1$ (columns 4 and 7). We also see that the applicants in panel

A diversify their applications by including programs in more fields and changing their top ranked program to another field. For applicants in panel B, we see that they become more confident of being accepted to highly ranked programs in their applications, so they reduce the length of their applications. Further, we also see that a few of them change the top-ranked programs and fields in their applications, likely as they have more favorable beliefs about being accepted to other programs.

Table 11: Policy changes to capacities: Characteristics of accepted programs

	Baseline	No updating			Updating		
γ (Fraction of original capacity)	1	0.9	0.5	0.1	0.9	0.5	0.1
Panel A: Humanities							
D	8.30	-0.05	-0.08	-0.22	-0.07	-0.09	-0.12
G	0.51	0.04	0.17	-0.20	0.01	0.00	-0.40
U	7.24	-0.13	-0.85	-2.25	-0.21	-1.14	-3.01
$\overline{w}$	1.88	0.02	0.16	0.44	0.04	0.22	0.60
σ_w	1.04	0.00	0.02	0.07	0.01	0.03	0.09
Panel B: All other fields							
D	7.89	0.01	0.01	0.00	-0.01	0.02	0.01
G	0.44	-0.03	-0.13	-0.24	-0.04	-0.14	-0.21
U	3.67	-0.03	-0.12	-0.18	0.00	-0.12	-0.17
$\overline{w}$	2.65	0.01	0.03	0.04	0.00	0.03	0.04
σ_w	1.12	0.00	0.00	0.00	0.00	0.00	0.00
Observations	12,964						
Simulations	2						

Note: Column 1 ($\gamma = 1$) contains the baseline application characteristics and outcomes of the matching mechanism. Panel A is for applicants with a program in Humanities as their top rank in the baseline setting, and panel B is for applicants with other programs as their top rank in the baseline setting. The numbers of columns 2 to 7 are all expressed as row-wise deviations from column 1.

Table 11 shows the overall counterfactual characteristics of the programs where applicants are accepted. The table layout is the same as in table 10. The table contains two panels, A and B, and the applicants in panels A and B are the same across the columns. Further, columns 2-4 show the program characteristics compared to the baseline for the setting with no updating, and columns 5-7 show the program characteristics compared to the baseline for the setting with updating. I first compare the baseline characteristics of programs applicants are accepted to for the two panels, column 1 in table 11. We see that if we compare panels A and B in the baseline setting, applicants with a program in humanities as their top rank (panel A) are accepted into programs that are further away, have peers with lower relative GPAs', can expect around four months of additional unemployment, and almost $10,000$ DKK lower monthly starting wages on average. Further, applicants in panel A also have a slightly lower dispersion in expected starting wages.

In columns 2-4 of table 11, the setting with no updating, we see that, as we decrease capacities for programs in humanities, applicants in panel A are accepted to programs further away, with lower expected unemployment and higher expected monthly starting wages. This is somewhat counter intuitive, but the explanation is that the programs in humanities are also the programs with the highest expected unemployment and the lowest expected starting wages on average. So when we reduce capacities for programs in humanities, many of the applicants in panel A are accepted into programs in other fields. We do not see the same pattern for applicants in panel B, where the characteristics of accepted programs do not change much.

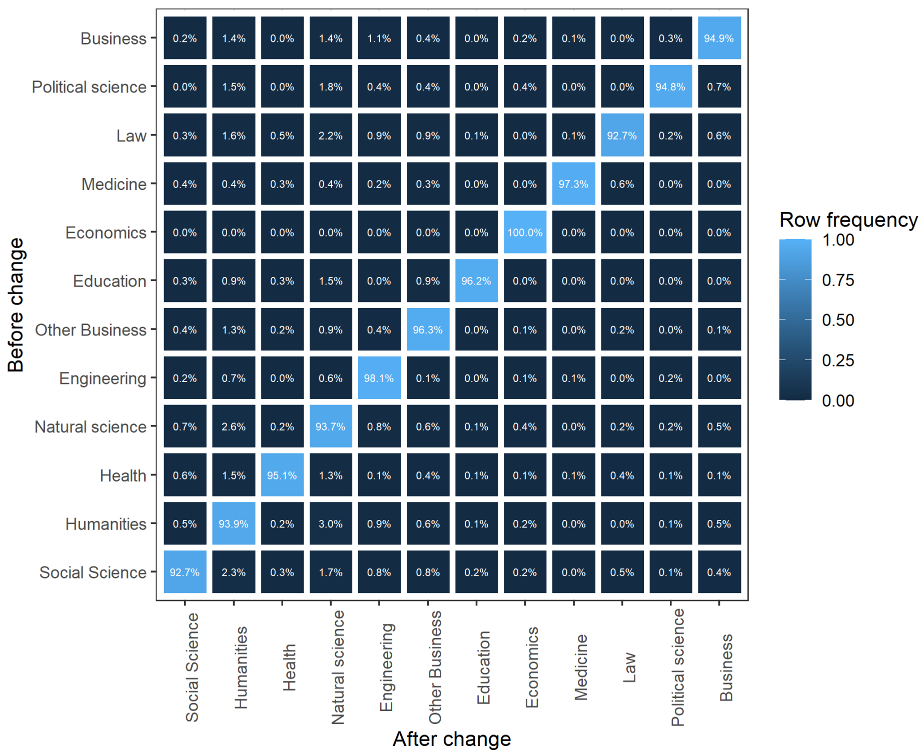
When I instead look at columns 5-7 in table 11, which are for the setting where applicants can update their beliefs, I see a similar pattern as in columns 2-4. However, the magnitude of the changes in characteristics is larger for applicants in panel A. In columns 2-4, where applicants cannot update their beliefs, the difference in program characteristics is bounded by the applications in the baseline setting. This is not the case when I allow them to update their beliefs.

So from tables 10 and 11, I see that when I reduce capacities for programs in humanities while increasing the capacities of other programs correspondingly, applicants who had a program in humanities as their top rank before the change overall see the biggest change. If I do not allow them to update their beliefs in response to the changes, they are less likely to be accepted to a program. However, if they are accepted to a program, the program is closer and has lower expected unemployment and higher expected starting wages. When I, on the other hand, allow them to update their beliefs. In that case, this attenuates the drop in the probability of being accepted, as the applicants can include other programs in their applications, where they are more likely to be accepted. A result of the change in application patterns is that the programs where the applicants in panels A and B in table 11 are accepted are much more similar.

9.4 Application patterns

Figure 5 shows the distribution of top ranked programs by field before (second axis) and after the policy change (first axis) for a 10% reduction in capacities for programs in humanities ($\gamma = 0.9$). The figure illustrates how changes to the capacities affect the application behavior when applicants are allowed to update their beliefs. The case where applicants cannot update their beliefs is trivial, and I do not report it, as applicants do not change their behavior in this case. I first look at applicants with a program in humanities as their top rank (the second to last row) before the change. We see that after the reduction in capacities for the programs in humanities, 93.9% of them still include a program in humanities as their top rank. Interestingly, most applicants either move to natural sciences, social science, or engineering programs. A possible explanation for this pattern is that the applicants who move away from programs in humanities are most likely at the lower end of the ability distribution. As programs in natural sciences typically face low demand and hence have low GPA cutoffs or free slots even applicants

Figure 5: Distribution of top-ranked program by field before and after policy change, $\gamma = 0.9$ and applicants update beliefs



Note: The second axis indicates top ranked field before the change, and the first axis indicates top ranked field after the change. The cells are color coded by the share of applications with a given before/after field combination by field. In other words, the percentages on each row sum to 100%. The policy parameter γ gives the fraction of capacities in Humanities which are left, and conversely, $1 - \gamma$ gives the fraction of capacities in Humanities, which are redistributed across the other fields.

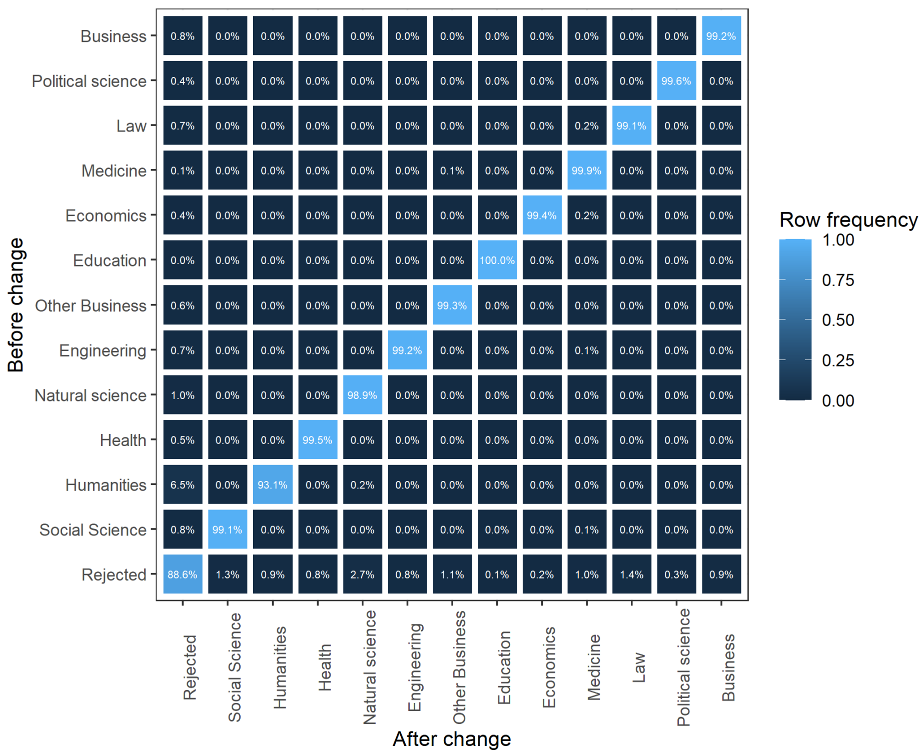
with low ability have relatively high beliefs about being accepted in them. Further, as they also lead to expected wages in the top end and a low expected unemployment these programs are attractive to these applicants. If we turn to the other rows of figure 5, we see that it is not just applicants who had a program within humanities as their top rank who changed their top ranks. Some of the applicants with programs in other fields also change their top ranks, even though these programs all have increased capacities. This might seem puzzling at first, but as the offers arise in the equilibrium, where the algorithm matches applicants and programs according to their preferences, changes to capacities will affect the application behavior in my model for applicants who are not directly affected by, e.g., a reduction. Further, the figure only reports the top ranked program, and the affected applicants might just have ranked their previous top ranked program lower in their application. Appendix figure 9 where I have reduced the capacities for programs in humanities by 50% ($\gamma = 0.5$) shows a similar pattern to figure 5. However, the share of applicants who had a program in humanities as their top rank before the change is even lower, and the share of applicants with programs in other fields as top rank before the change changes less than in figure 5. I can explain the last part by the fact that all other programs now have even more capacities.

9.5 Accepted programs

Next, I look at where applicants are accepted. I first look at where applicants are accepted when they cannot update their beliefs, figure 6, before I look at where applicants are accepted, when they can update their beliefs, figure 7. Figure 6 shows the distribution of programs where applicants are accepted by field, before (second axis) and after the policy change (first axis) for a 10% reduction in capacities for programs in humanities ($\gamma = 0.9$) where applicants cannot update their beliefs. Compared to figure 5, figure 6 has an additional row and column for applicants who are rejected from all programs in their applications. Further, when the applicants cannot update their beliefs, the applications are fixed while the capacities change. Most of the rows in figure 6 are not so interesting as nearly all applicants accepted in a program in a given field before the change are also accepted in the same field after the change. The only two rows where something happens are the third last row (humanities before change) and the last row (applicants rejected before change). We see that 6.5% of applicants accepted to a program in humanities before the change are rejected, and close to 0% are accepted in programs in other fields. In the last row, we see that the share of applicants who were rejected before the change is reduced by 11.4%.

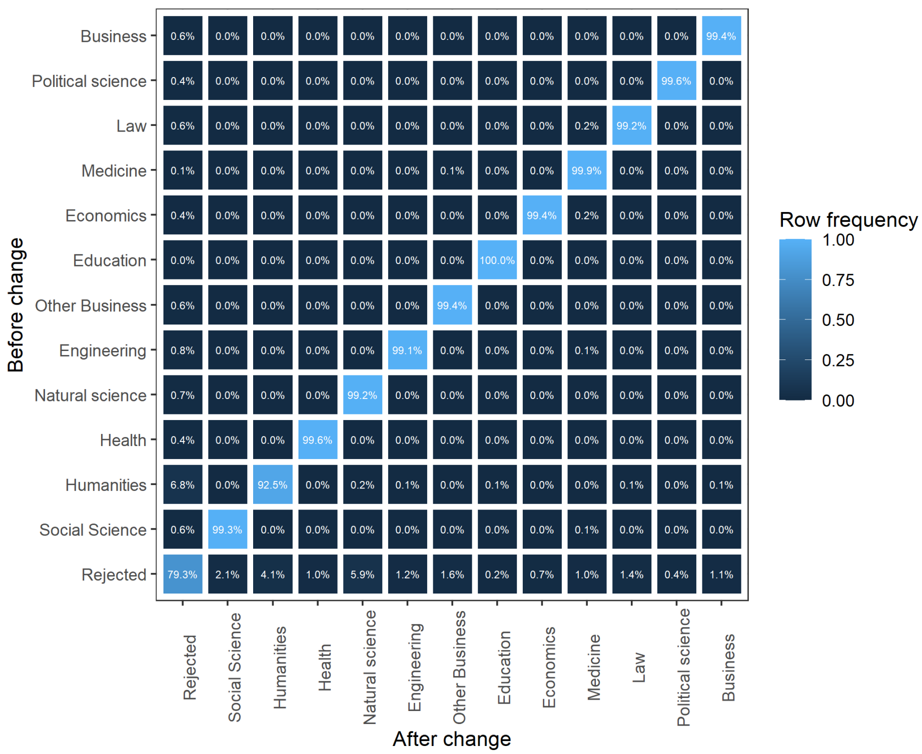
Figure 7 shows the distribution of programs where applicants are accepted by field, before (second axis) and after the policy change (first axis) for a 10% reduction in capacities for programs in humanities ($\gamma = 0.9$) and applicants can update their beliefs. As figure 6, figure 7 also has an additional row and column for applicants who are rejected from all programs in their applications. Allowing applicants to update their beliefs according to the change in capacities creates a larger shift in which program

Figure 6: Distribution of accepted program by field before and after policy change, $\gamma = 0.9$ and applicants cannot update beliefs



Note: The second axis indicates top ranked field before the change and the first axis indicates top ranked field after the change. The cells are color coded by the share of applications with a given before/after field combination within a before change field, in other words, the percentages on each row sum to 100%. The policy parameter γ gives the fraction of capacities in Humanities which are left, and conversely $1 - \gamma$ gives the fraction of capacities in Humanities, which have been redistributed across the other fields.

Figure 7: Distribution of accepted program by field before and after policy change, $\gamma = 0.9$ and applicants can update beliefs



Note: The second axis indicates top ranked field before the change, and the first axis indicates top ranked field after the change. The cells are color coded by the share of applications with a given before/after field combination within a before change field. In other words, the percentages on each row sum to 100%. The policy parameter γ gives the fraction of capacities in Humanities which are left, and conversely, $1 - \gamma$ gives the fraction of capacities in Humanities, which have been redistributed across the other fields.

applicants are accepted, although as we see in figure 7, it is again only applicants who were accepted to a program in humanities or rejected from all programs before the change who are accepted to other programs or rejected. In the third to last row, we see that applicants who were accepted to a program in humanities before the change are now slightly more likely rejected than in figure 6. Further, we see that previously rejected applicants in the last row are now more likely to be accepted compared to figure 6. The model prediction that more applicants who previously were accepted to a program in humanities are rejected from all programs in the updating setting (figure 7) than in the no updating setting (figure 6) when I change the capacities is unexpected in the sense that I would expect them to include other programs with a better chance of being accepted (increased capacities). The likely mechanism behind this is that applicants with low GPAs are most likely to change their applications. Since programs in humanities have many applicants with low GPAs, these applicants will also be at the bottom of the distribution within other fields and, therefore, more likely rejected, even though the capacities of other programs have increase. In appendix figure 11 I reduce the capacities by 50% ($\gamma = 0.5$) instead of 90% in figure 7. I see the same pattern in figure 11; it is mostly applicants who were accepted to programs in humanities or rejected from all programs which are affected by the change in capacities. The effect is larger than in figure 7 and not one-to-one with the change in capacities.

To sum up, the reduction to capacities in humanities, and conversely an increase in all other programs' capacities, causes applicants in all groups to change their top ranked fields for small changes in capacities. However, for larger changes in capacities, it is mainly applicants who had a program in humanities as their top rank before the change who are affected, although the change is less than one-to-one with the change in capacities. For small changes, I explain this by excess capacities in programs in humanities. For large changes in capacities, I interpret it as applicants in humanities having strong preferences. Further, we also see that mainly applicants who were accepted to a program in humanities or rejected from all programs before the change are accepted to programs in other fields or rejected after the change. This holds for the setting where applicants cannot update their beliefs and the setting where they can update their beliefs.

10 Conclusion

In this paper, I have studied the effect of changes to supply, through changes to capacities, for higher education programs on the demand. I have in particular studied a "neutral" reduction in capacities for programs in humanities, and found that applicants in general respond to the change, when they can update their beliefs accordingly. Further, I found that the response is not one-to-one with the changes to capacities, especially for applicants who applied to programs in humanities before the change, as they have strong preferences for these programs. Lastly I found that making capacities and changes to capacities public before applicants submit their applications can help applicants who otherwise would be rejected from all programs in their applications.

A straightforward extension to the current model is to extend it to the full universe of applicants. I plan to do this at a later stage and it requires information on the characteristics of the universe of applicants and programs.

Another more challenging extension would be to include the extensive margin of applications to the model as an outside option. This requires including a first step in the model, where applicants choose between the option to apply or not. This allows the study of how the extensive margin of applicants is affected by changes to capacities.

Finally, a very challenging extension to the current model is incorporating the education market's supply side into the model. This requires modeling of the decisions by universities on how to spend their funds. This allows one to look at the full equilibrium effects of changes to capacities and the effect of making universities declare their available capacities publicly before applicants submit their final applications. Such a model would make it possible to understand how not just applicant demand but also universities would respond, e.g., by looking at the trade-off between quality and quantity in teaching and the decision of time allocation of faculty for teaching and research.

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A Linking program identifiers in the application data to DST registers

There is no direct link between the program identifiers in the application data and the program identifiers in the register data, which are a combination of education codes (UDD) and institution codes (INTSNR). I, therefore, have to create one. Before I describe how I generate the link, it is useful to describe a few difficulties I must address to ensure the link is reliable. There are mainly two issues that prevent me from just making a simple merge of applicants from the application data to the register data by personal identification numbers to see which education and institution codes they have in the register data, when they have gotten an offer in the application data. Firstly, some program identifiers in the application data cover multiple program identifiers in the register data and the other way around. Secondly, I can only observe offers in the application data, not which program the applicant enrolls in and the other way around in the register data. To solve the problem, I partly rely on a simple algorithm that, for each program identifier in the application data, finds the share of applicants with an offer who have enrolled in a program in the register data in the current application cycle. I then say a program combination is linked when a sufficiently high share with an offer for a program in the application data is enrolled in a program in the education registry. I further also condition on programs with a sufficiently high number of applicants with an offer.

In practice, I rely solely on the algorithm for programs with a share $x \geq 95\%$ and at least 100 applicants with an offer in the application data. For the remainder of the programs, I define the links by hand, using the share and number of students to guide me.

There is one further difficulty, the education register only contains students who are enrolled in a program by the 1st of October. Therefore it does not contain early dropouts. This should not be a big problem, but it is conceivable that the tendency to drop out early varies across programs, and further, the problem increases in magnitude as the uptake of a program gets smaller. There is, unfortunately, not much I can do to solve this problem, although I try to mitigate it by the heuristic that programs with smaller uptakes should have a higher share enrolled in a given education identifier.

Using this combined approach, I can identify combinations of education and institution codes from the registry data for 582 out of 897 program identifiers in the application data.

B List of example programs within fields

Table 12 contains lists of examples of education programs within the fields I have defined.

Table 12: Examples of programs in the defined fields

Field	Example programs
Social science	Sociology (KU), Anthropology (AU), Psychology (SDU), Psychology (AAU)
Humanities	English (AU), Danish (SDU), Danish (KU), History (AAU)
Health	Dentistry (KU), Sports Science (KU), Dentistry (AU), Musical Therapy (AAU)
Natural science	Math (AU), Biology (KU), Computer Science (SDU), Physics (KU)
Engineering	Civ. Eng. Energy (AAU), Civ. Eng. Elektronics (DTU), Civ. Eng. Nanotechnology (AAU)
Other Business	Business Law (CBS), International Business (CBS), Marketing and Management Com. (AU), Business Law (AAU)
Education	Pedagogy (KU), Speach Therapy (SDU), Audiology (SDU)
Economics	Economics (KU), Economics (SDU), Economics (AU), and Economics (AAU)
Medicine	Medicine (KU), Medicine (SDU), Medicine (AU), and Medicine (AAU)
Law	Law (KU), Law (AU), Law (AAU), Law (SDU)
Political science	Political science (KU), Political science (SDU), and Political science (AU)
Business	Business Economics (CBS), Business Economics (SDU), Business Economics (AU)

C Belief updating

The following algorithm updates applicants' beliefs. The algorithm is based on algorithm 2 in Larroucau and Rios (2022). Larroucau and Rios (2022) use it to update beliefs as students update the information they have about their abilities in under the different policies they evaluate. I have slightly adapted the algorithm to update beliefs given new program capacities.

Algorithm 1 Updating beliefs

Input: $\hat{\theta}$, $\hat{p}^0$, ROL^{Full}, C_{old}, C_{new}, $score$, ϵ_{tol}
Output: p, P
  for `each s in S` **do**
    Solve Portfolio Choice problem to get $ROL^{Analysis}$ given $(\hat{\theta}, \hat{p}_s^0)$
    Substitute applicants in $ROL^{Analysis}$ into ROL^{Full}
    Bootstrap cutoff distribution to get P^0 given (ROL^{Full},$score$,C_{old})
    Estimate $\hat{\delta}^0 \equiv (\hat{\mu}^0, \hat{\sigma}^0)$
  end for
  Stack $\hat{\delta}^0$ over simulations S
  $\delta_{diff} = 2\epsilon_{tol}$, $k = 1$, $\rho = 0.9$
  while $\delta_{diff} > \epsilon_{tol}$ **do**
    for `each s in S` **do**
      Solve Portfolio Choice problem to get $ROL^{Analysis}$ given $(\hat{\theta}, p_s^{k-1})$
      Bootstrap cutoff distribution to get $\tilde{P}_s^k$ given (ROL^{Full},$score$,C_{new})
      Estimate updated beliefs $\hat{p}_s^k$
      Take point-wise convex combination of cutoffs $\hat{P}_s^k = \rho^k \hat{P}_s^{k-1} + (1-\rho^k)\tilde{P}_s^k$
      Estimate $\hat{\delta}_s^k \equiv (\hat{\mu}_s^k, \hat{\sigma}_s^k)$
    end for
    Stack $\hat{\delta}^k$ over simulations S

    Compute $\delta_{diff} = \|\hat{\delta}^k - \hat{\delta}^{k-1}\|$, $\hat{p} = \hat{p}^{k-1}$, $k++$
  end while

$\hat{\theta}$ are the estimated preference parameters, ROL^{Full} is the submitted preference ordering for the full sample, $ROL^{Analysis}$ is the preference ordering for the *analysis* sample, C_{old} is the old program capacities, C_{new} is the new program capacities, $score$ is a vector holding applicant scores, S is the number of simulations, μ is the mean of the cutoff distribution, σ is the standard deviation of the cutoff distribution, p is a matrix containing applicants estimated beliefs, and P is a matrix containing the simulated cutoff distribution for all programs.

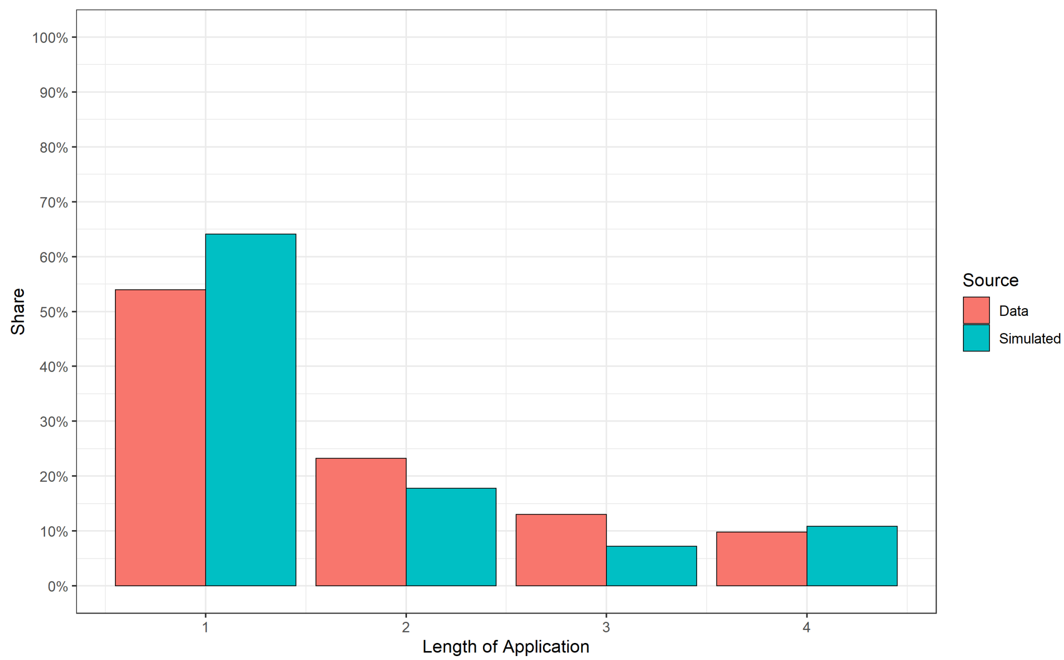
D Additional tables and figures

Table 13: Characteristics of programs in applications by rank

	Rank 4 Mean/Std	Rank 5 Mean/Std	Rank 6 Mean/Std	Rank 7 Mean/Std	Rank 8 Mean/Std
Distance (10 Km)	10.36	9.33	10.16	8.81	10.75
	9.48	8.90	9.07	8.33	8.26
Standardized GPA	-0.36	-0.41	-0.51	-0.38	-0.27
	1.75	1.89	2.02	2.08	1.86
Unemployment (Months)	4.66	4.94	5.13	5.33	5.49
	2.81	2.77	2.81	2.55	3.42
Expected earnings (10,000 DKK)	2.48	2.47	2.45	2.43	2.36
	0.59	0.56	0.56	0.57	0.55
Dispersion of Expected earnings (10,000 DKK)	1.11	1.17	1.18	1.27	1.05
	0.44	0.47	0.49	0.63	0.18
Observations	1,269	494	222	102	53

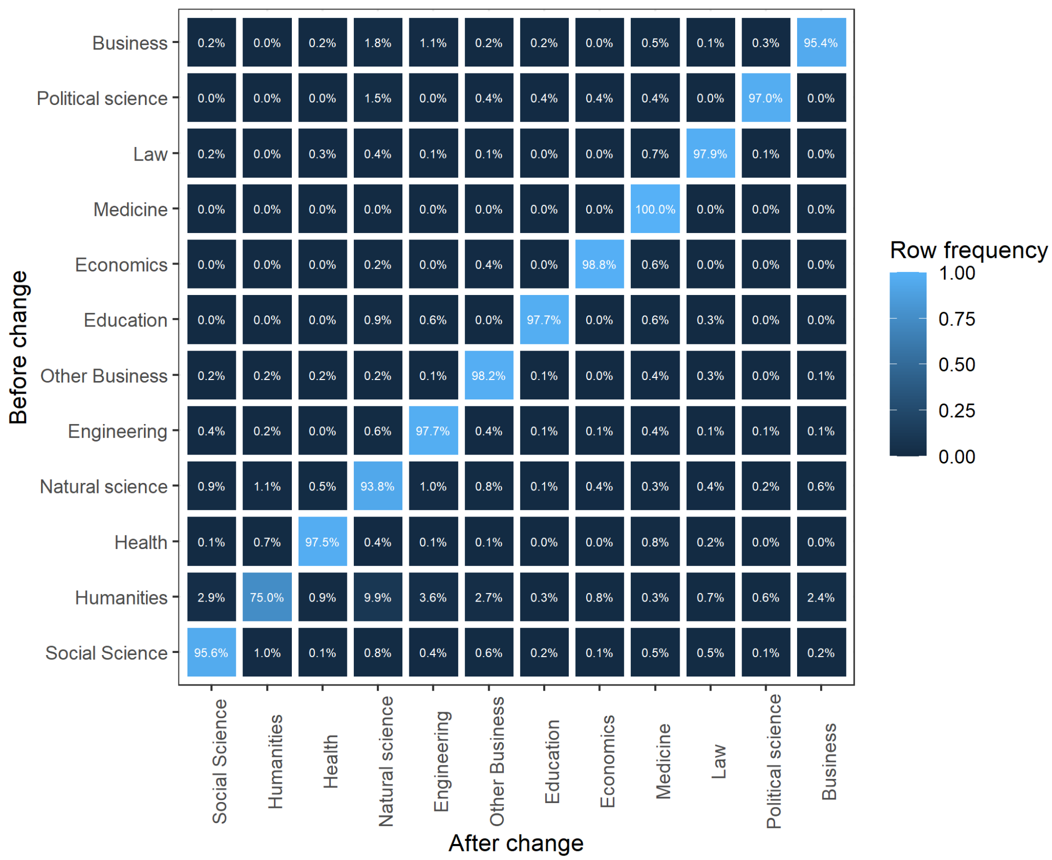
Note: The reported numbers are means (standard deviations in parentheses). The first column reports variable names and units in parentheses. The columns indicate for which rank in the applications the measures are. The number of observations shows the number of applicants with at least the given number of ranks in their application.

Figure 8: Empirical and simulated share of applicants by the number of programs in their application using beliefs from additional step



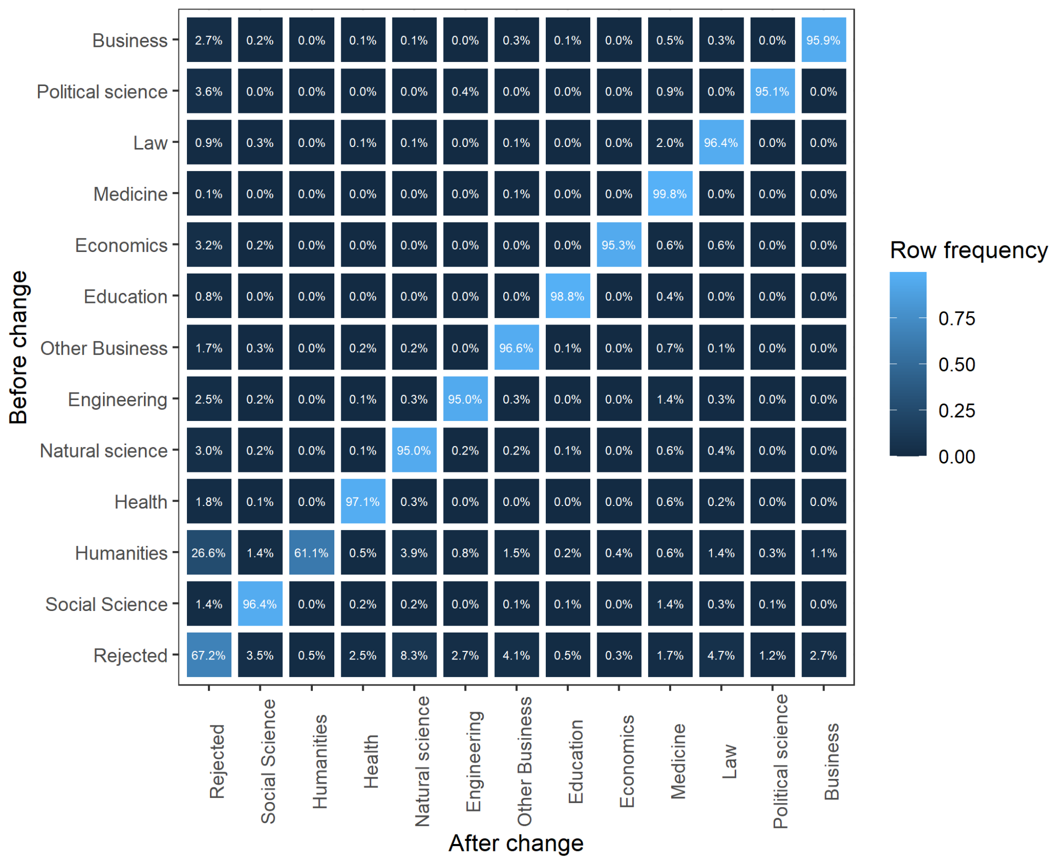
Note: The first axis displays the number of programs in an application (length of the application) and the second axis displays the share of applicants in percentages. The color of the bars indicates the source, red represents the empirical distribution and green indicates the simulated distribution.

Figure 9: Distribution of top ranked program by field before and after policy change, $\gamma = 0.5$ and applicants update beliefs



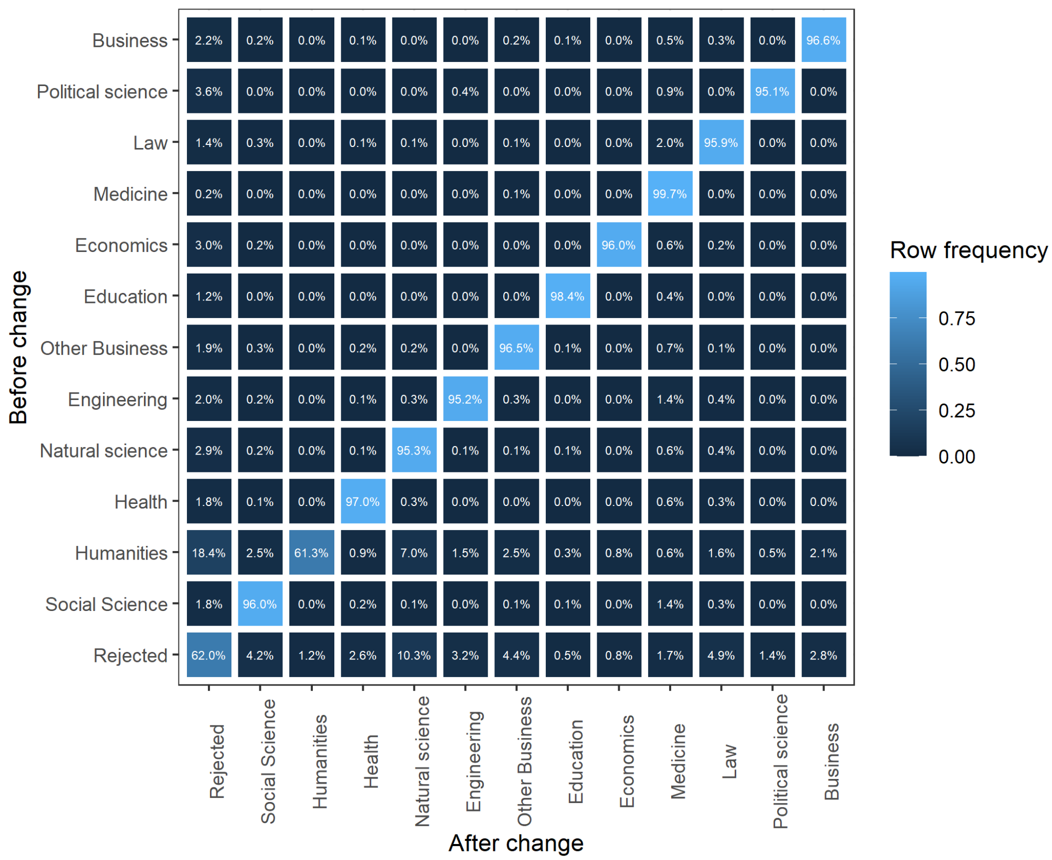
Note: The figure shows the distribution of top ranked programs by the field before and after the policy change. The second axis indicates top ranked field before the change and the first axis indicates top ranked field after the change. The cells are color coded by the share of applications with a given before/after field combination within a before change field, in other words, the percentages on each row sum to 100%. The policy parameter γ gives the fraction of capacities in Humanities which are left, and conversely $1 - \gamma$ gives the fraction of capacities in Humanities, which have been redistributed across the other fields.

Figure 10: Distribution of top ranked program by field before and after policy change, $\gamma = 0.5$ and applicants cannot update beliefs



Note: The figure shows the distribution of top ranked programs by the field before and after the policy change. The second axis indicates top ranked field before the change and the first axis indicates top ranked field after the change. The cells are color coded by the share of applications with a given before/after field combination within a before change field, in other words, the percentages on each row sum to 100%. The policy parameter γ gives the fraction of capacities in Humanities which are left, and conversely $1 - \gamma$ gives the fraction of capacities in Humanities, which have been redistributed across the other fields.

Figure 11: Distribution of top ranked program by field before and after policy change, $\gamma = 0.5$ and applicants update beliefs



Note: The figure shows the distribution of top ranked programs by the field before and after the policy change. The second axis indicates top ranked field before the change and the first axis indicates top ranked field after the change. The cells are color coded by the share of applications with a given before/after field combination within a before change field, in other words, the percentages on each row sum to 100%. The policy parameter γ gives the fraction of capacities in Humanities which are left, and conversely $1 - \gamma$ gives the fraction of capacities in Humanities, which have been redistributed across the other fields.